Equipe MGII

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Estimating Criteria for Fitting B-spline Curves: Application to Data Compression

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Outline

- Survey of the present compression methods
  - Strategies based on polygonal curves
  - Strategy based on spline curves
- A new compression strategy
  - Data fitting with B-splines
  - Criteria for estimating data approximation
  - Reduction technique
  - Result comparison
- Conclusion

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Survey of the present methods

Stategies based on polygonal curves

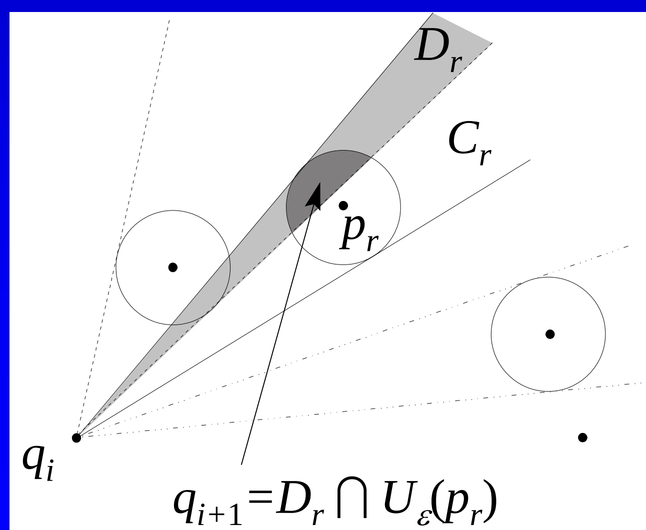
Inputs ⟹ A polygonal curve $P = (p_0, \ldots\ldots, p_n)$

⟹ A tolerance $\varepsilon \geq 0$

Output ⟹ A polygonal curve $Q = (q_0, \ldots\ldots, q_m)$ with $m < n$ such as $d(P,Q) \leq \varepsilon$

- **The intersecting cones method** (E. Arge & M. Daehlen)

General strategy



Intersection of cones originated at p_i

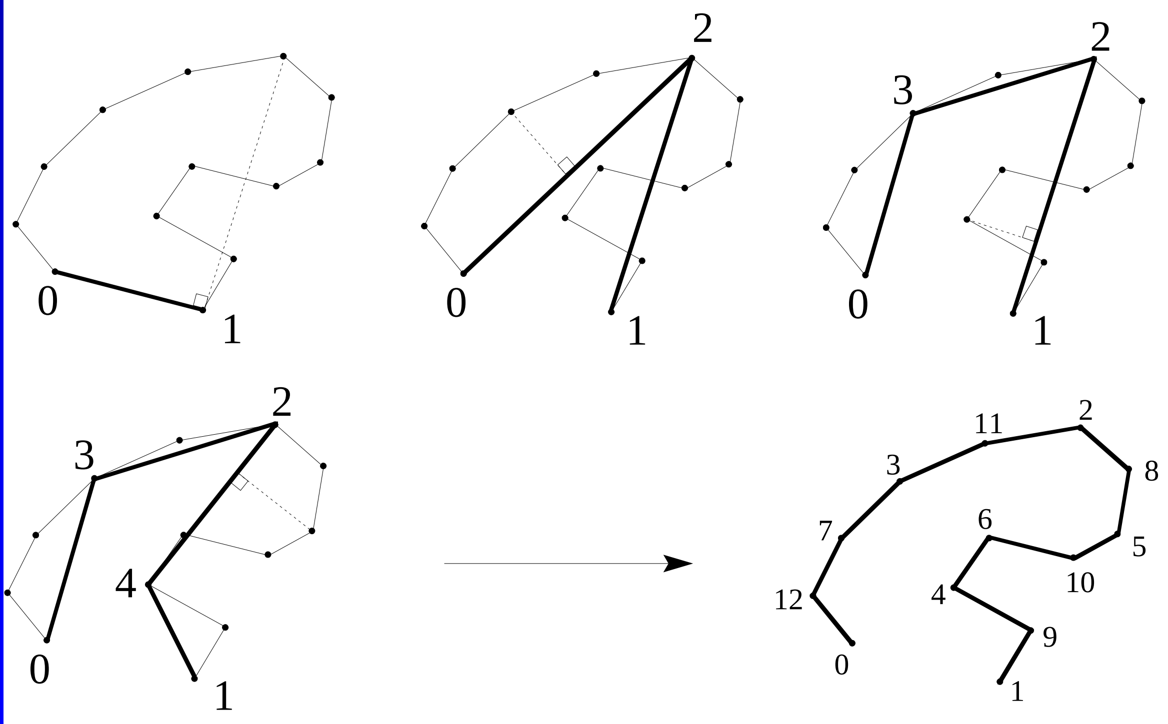
Survey of the present methods

- **Douglas and Peucker's method without tolerance**

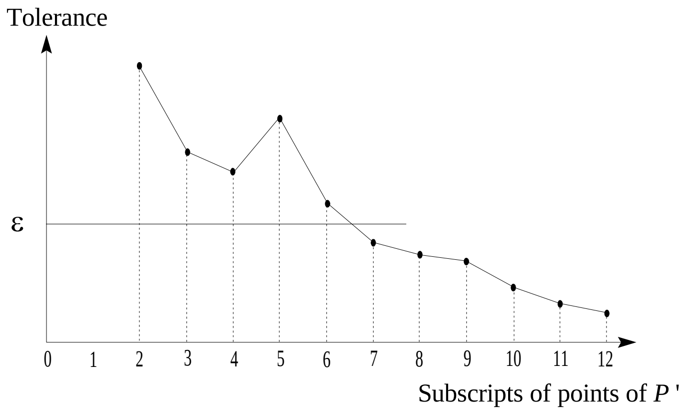
General strategy

Rearrange the points of P

$$P = (p_0, \ldots\ldots, p_n) \longrightarrow P' = (p'_0, \ldots\ldots, p'_n)$$



Multi-scale analysis



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Survey of the present methods

Stategies based on spline curves

The knot removal strategy of T. Lyche & K. Morken

Inputs
- A polygonal curve $P = (p_0, \ldots\ldots, p_n)$
- A tolerance $\varepsilon \geq 0$
- An interpolating B-spline f on $T = (t_i)_{i=0}^{n+k}$

Output
- An approximating B-spline g $\tau = (\tau_i)_{i=0}^{m+k}$ with m < n and $\tau \subset T$ such as $\|f - g\|_* \leq \varepsilon$

General strategy

+ Computation of weights $\omega_i = \|f - g^i\|_*$
+ Selection of knots to be removed

  t_i can be removed if $\omega_i \leq \varepsilon$
+ Reconstruction of the approximating curve

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Survey of the present methods

The knot removal strategy of M. Eck & J. Hadenfeld

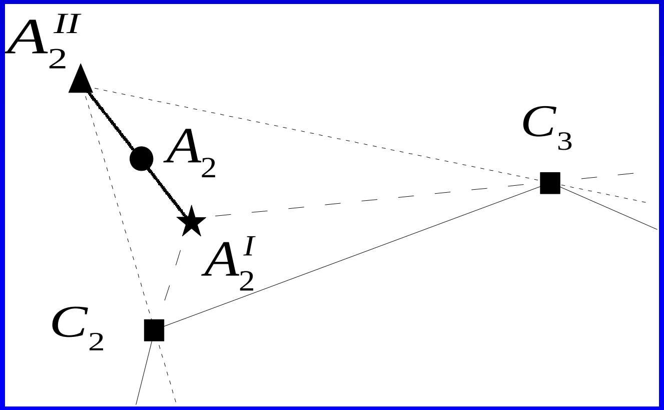
Inputs / Output ⇒ The same as in T. Lyche and K. Morken's strategy

where $f(t) = \sum_{i=0}^{n} C_i N_{i,k,T}(t)$ is an interpolating B-spline

General strategy

Removal of knot t_i ⇒ Approximating Bspline g^i on $\mu = T - \{t_i\}$

$$g^i(t) = \sum_{j=0}^{n-1} A_j N_{j,k,\mu}(t)$$



"forward" construction of g_I^i with A_j^I

"backward" construction of g_{II}^i with A_j^{II}

+ Position of control point A_j in $[A_j^I, A_j^{II}]$

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A new compression strategy

Data fitting with B-splines

Problem

Given a polygonal curve $P = (p_0, \ldots\ldots, p_n)$, find a curve $f(t) = \sum_{i=0}^{m} Q_i N_{i,k,T}(t)$ as close as possible of P

Least squares fitting: find control points Q_j so that

$$\sum_{j=0}^{n} \left(f(\zeta_j) - p_j \right)^2$$ is minimum → **Householder**

Choice →
- **parameters ζ_i**
- **knot vector T**
- **order k**

A new compression strategy

Choice of a parameterization method

- **uniform** $h_i = \text{constant}$
- **cumulative chord length** $h_i = \left\| p_{i+1} - p_i \right\|$
- **centripetal** $h_i = \sqrt{\left\| p_{i+1} - p_i \right\|}$

General expression proposed by Lee

$$\zeta_0 = 0, \quad \zeta_i = \frac{\sum_{j=0}^{i} \left\| p_j - p_{j+1} \right\|^e}{\sum_{j=0}^{n} \left\| p_j - p_{j+1} \right\|^e} \quad (e \leq 0 \leq 1)$$

- uniform with $e = 0$
- cumulative chord length with $e = 1$
- centripetal with $e = 0.5$

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A new compression strategy

Choice of a knot vector

- **uniform** constant spacing
- **from an extension of De Boor formula for interpolation**

$$\begin{cases} t_0 = \ldots\ldots = t_{k-1} = \zeta_0 \\ t_{m+1} = \ldots\ldots = t_{m+k} = \zeta_n \\ t_{i+k} = \dfrac{\zeta_{i+l_1} + \ldots\ldots + \zeta_{i+l_2}}{l_2 - l_1 + 1} \end{cases}$$

for $i = 0, \ldots\ldots, m\text{-}k$

Result

Best results in approximation with

a knot vector from the extension of De Boor formula

a centripetal parameterization

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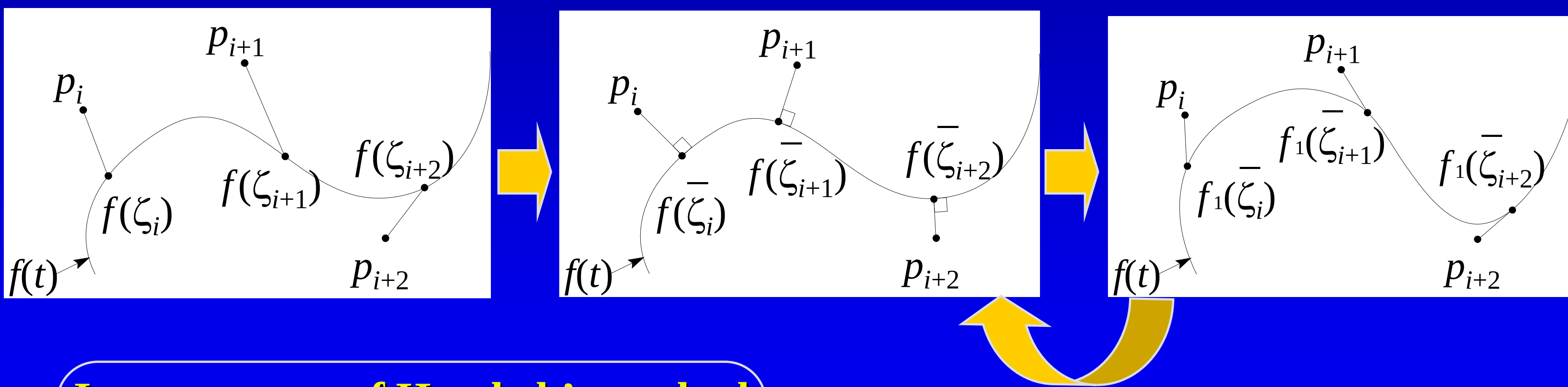
A new compression method

Two other parameterizations

- **Foley and Nielson's parameterization** : geometrical properties (angles)
- **Intrinsic Hoschek's parameterization**



Improvement of Hoschek's method

- ☑ Best global approximation
- ☑ Best convergence speed

Common problem

- ☑ Oscillation problem

A new compression method

Criteria for estimating data approximation

Definitions of norms : N_∞ et N_2

$$f(t)=\sum_{i=0}^{m} Q_i N_{i,k,T}(t)$$ is the fitting B-spline curve

- k is the order
- $T = (t_0,\ldots,t_{k-1},t_k,\ldots\ldots,t_m,t_{m+1},\ldots,t_{m+k})$ is the knot vector

f belongs to linear space $S_{k,T}$

Definitions

$$N_\infty(f)=Max\left\{Max\left\{\left|Q_{ji}\right| ; i=0,\ldots\ldots,m\right\}; j=1,\ldots\ldots,d\right\}$$

$$N_2(f)=\frac{\sqrt{\sum_{i=0}^{m}\left|Q_i\right|^2}}{(m+1)}$$

A new compression method

Polygonal curve

$g(t)=\sum_{i=0}^{n} R_i N_{i,2,\tilde{T}}(t)$ is the interpolating B-spline curve of initial data p_i

- order = 2
- $\tilde{T}=(\tilde{t}_0,\ldots,\tilde{t}_{k-1},\tilde{t}_k,\ldots\ldots,\tilde{t}_m,\tilde{t}_{m+1},\ldots,\tilde{t}_{m+k})$ is the knot vector

***g* belongs to linear space $S_{2,\tilde{T}}$**

To use N_∞ and N_2 on *f*-*g*, B-spline curve *f* and *g* should have

- **the same order**

  degree elevation (H. Prautzsch)

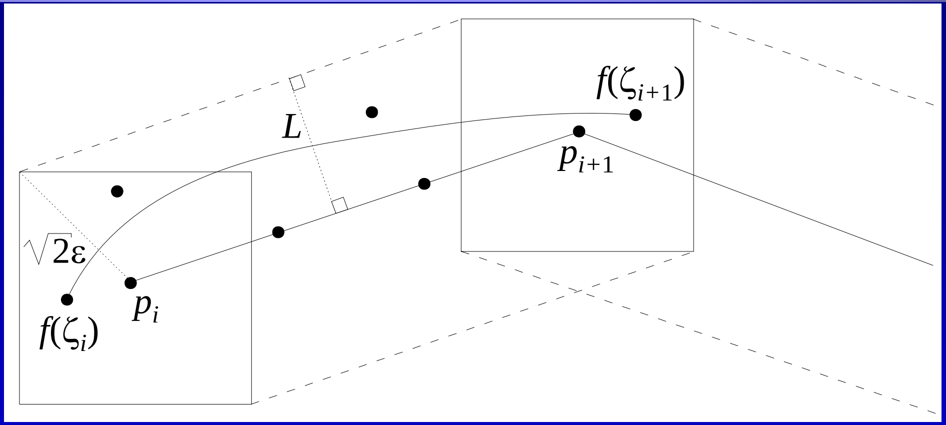
- **the same knot vector**

  subdivision algorithms (Boehm, Oslo, improved Oslo)

A new compression strategy

Result

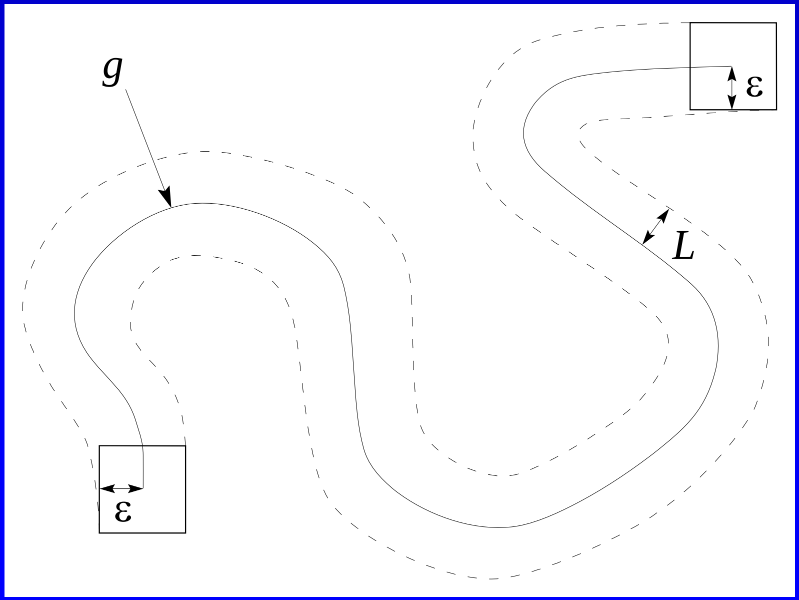
Band criterion for data approximation



$$N_\infty(f-g)=\varepsilon$$

Result

Band criterion for B-spline approximation



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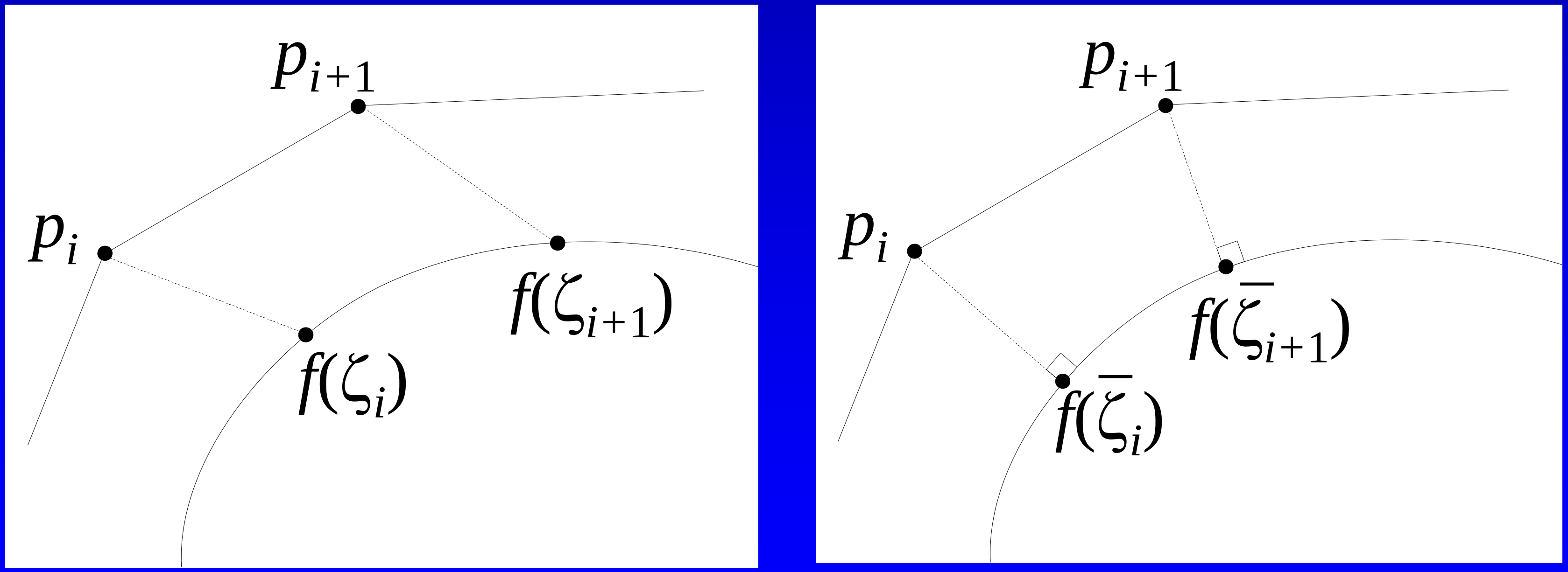
A new compression strategy

Local estimation criteria

Aim

To have a **local approach of approximation and estimation**

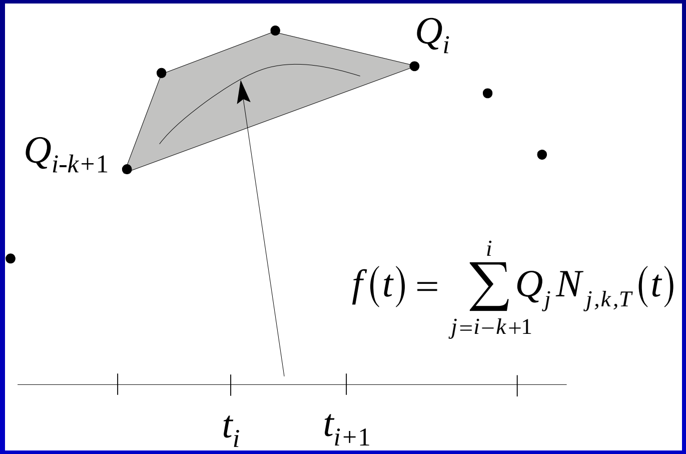
\+ **Find the best curve segment [p_i , p_{i+1}]**



A new compression strategy

+ **Use the convex hull of B-splines**

NOT ENOUGH

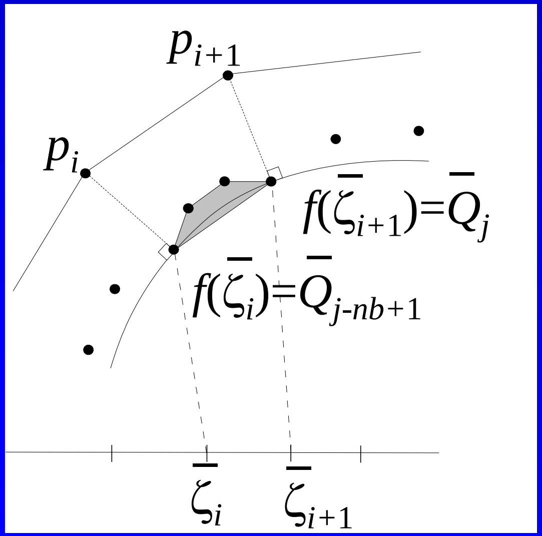


+ **Transform this best curve segment into its "Bézier" representation**

Apply the Hausdorff metric on both polygonal curves $P^i = (p_i, p_{i+1})$ and $\overline{Q}^j = (\overline{Q}_{j-nb+1,\ldots\ldots}, \overline{Q}_j)$

Result

If $d_H = (P^i, \overline{Q}^j) = \varepsilon$ then the best curve segment is at the most at ε distance from [p_i , p_{i+1}]



A new compression strategy

Reduction technique

General strategy

- Best choices
  - **Uniform knot vector**
  - **Intrinsic Hoschek's parameterization applied to a centripetal one**
- Principle
  - **A bissection method on the number of control points**

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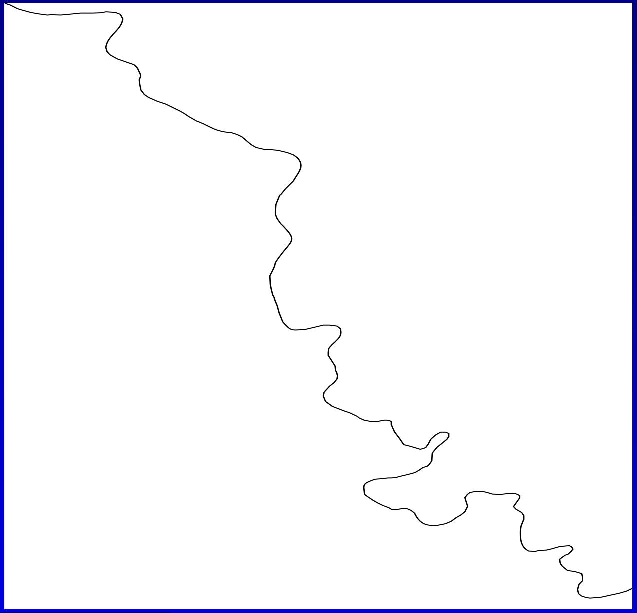
A new compression strategy

Results comparison

on compression rate

Initial isobathymetric line

Tolerance so that no visual difference



Results

Method	Knots	Strategy	Compression
Polygonal		Arge Daehlen	**68 %**
		Douglas Peucker	56 %
B-spline	De Boor	Lyche Morken	13 %
		Eck Hadenfeld	51 %
		Fitting (*e*=0.5)	22 %
	Uniform	Fittting (Hoschek *e*=0.5)	**61 %**
		Fitting (*e*=0.5)	49 %

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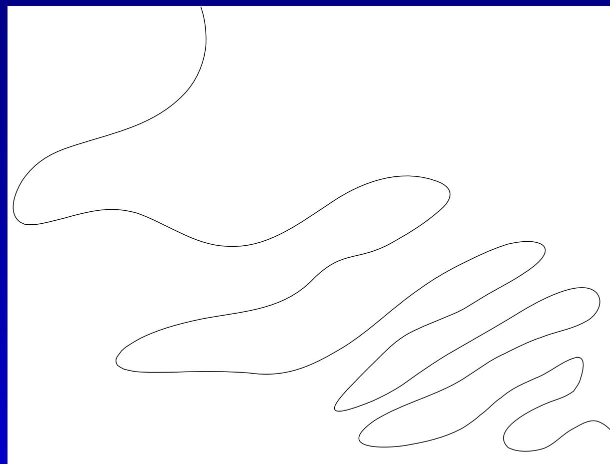
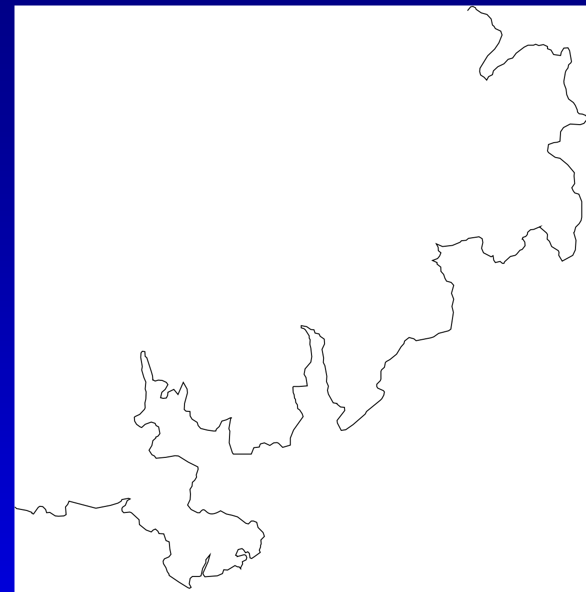
A new compression strategy



Initial smooth line



Initial complex line

Results

Method	Knots	Strategy	Compression (Smooth curves)	Compression (Complex curves)
Polygonal		Arge Daehlen	**74 %**	**61 %**
		Douglas Peucker	65 %	**51 %**
B-spline	De Boor	Lyche Morken	24 %	5 %
		Eck Hadenfeld	58 %	38 %
		Fitting (*e*=0.5)	34 %	11 %
	Uniform	Fittting (Hoschek *e*=0.5)	**68 %**	**49 %**
		Fitting (*e*=0.5)	59 %	40 %

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A new compression strategy

Results comparison

on computation time

Results

Method	Knots	Strategy	Computation time (Smooth curves)	Computation time (Complex curves)
Polygonal		Arge Daehlen	**0.1 ’’**	**2 ’’**
		Douglas Peucker	**0.4 ’’**	**3 ’’**
B-spline	De Boor	Lyche Morken	47’ 08’’	1h 22’’
		Eck Hadenfeld	40’ 31’’	59’ 21’’
		Fitting (*e*=0.5)	1’ 27’’	3’ 27’’
	Uniform	Fittting (Hoschek *e*=0.5)	9’ 33’’	13’ 24’’
		Fitting (*e*=0.5)	1’ 52’’	4’ 45’’

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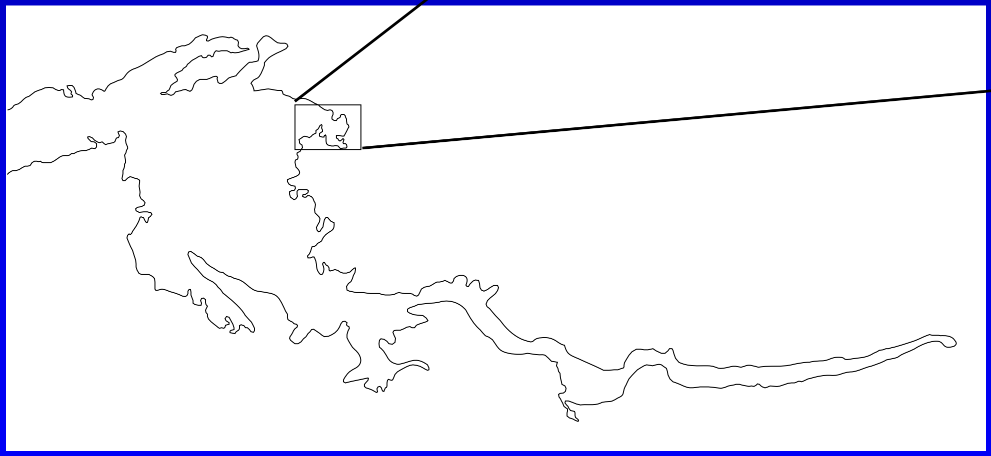
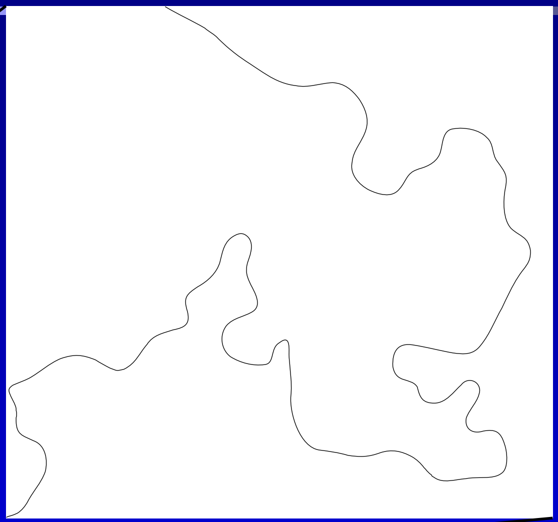
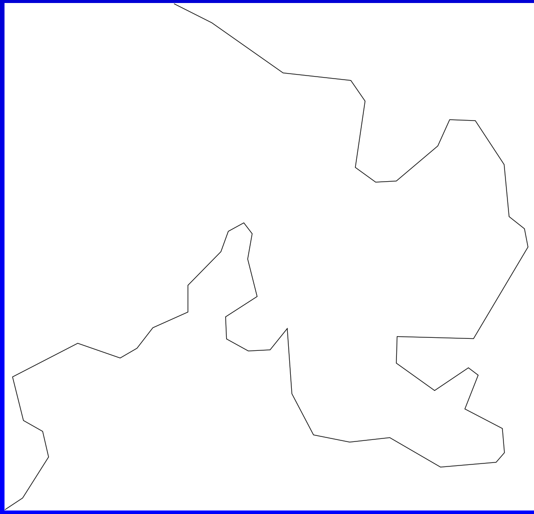
A new compression strategy

- **Advantage when**
  - **zooming in**







Useful in embarked cartographic information systems

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Conclusion

We have **build**

- **Norms N_∞ and N_2**

- **Local estimation criteria**

We have **improved**

- **Hoschek's technique**

Which has allowed us to **introduce**

- **A new reduction strategy**
  - Good reduction rates
  - Raisonable computation costs

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