

Nonlinear Color Interpretation of Physical Processes

D. V. Mogilenskikh

*Russian Federal Nuclear Center –
the All-Russian Scientific and Research Institute of
Technical Physics
named after academician E. I. Zababakhin
(RFNC-VNIITF)*

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1. Introduction

Color interpretation is one of the basic methods for graphical interpretation of physical processes.

The main points in the process of color interpretation are methods of generating a law of correspondence between physical characteristics and color space.

The main purpose: Development of algorithms for improving the informational content of images at color graphical interpretation of results of calculations and experiments.

Definition: A color palette is the ordered set of colors from RGB color space, which is represented with 1D array, in which each color corresponds to an integer-valued index.

Kinds of graphical presentation of palettes:

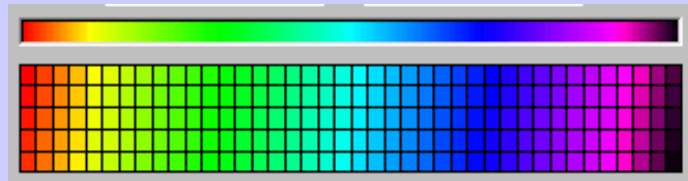


Fig. 1a

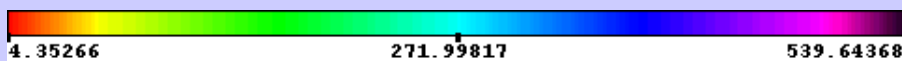


Fig. 1b

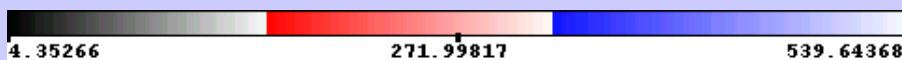


Fig. 1c

Let there be a palette of $(MC+1)$ -colors.

The description of the algorithms will be performed at the example of mathematical models specified by 2D regular difference grids.

The given assumption does not restrict the generality of application of algorithms for other data structures

We present the geometry of the initial data with 2D arrays of the coordinates $X[M, N]$, $Y[M, N]$, where $(M+1) \times (N+1)$ - is the grid dimensionality.

Let P denote one of the physical characteristics: $P[M-1, N-1]$.

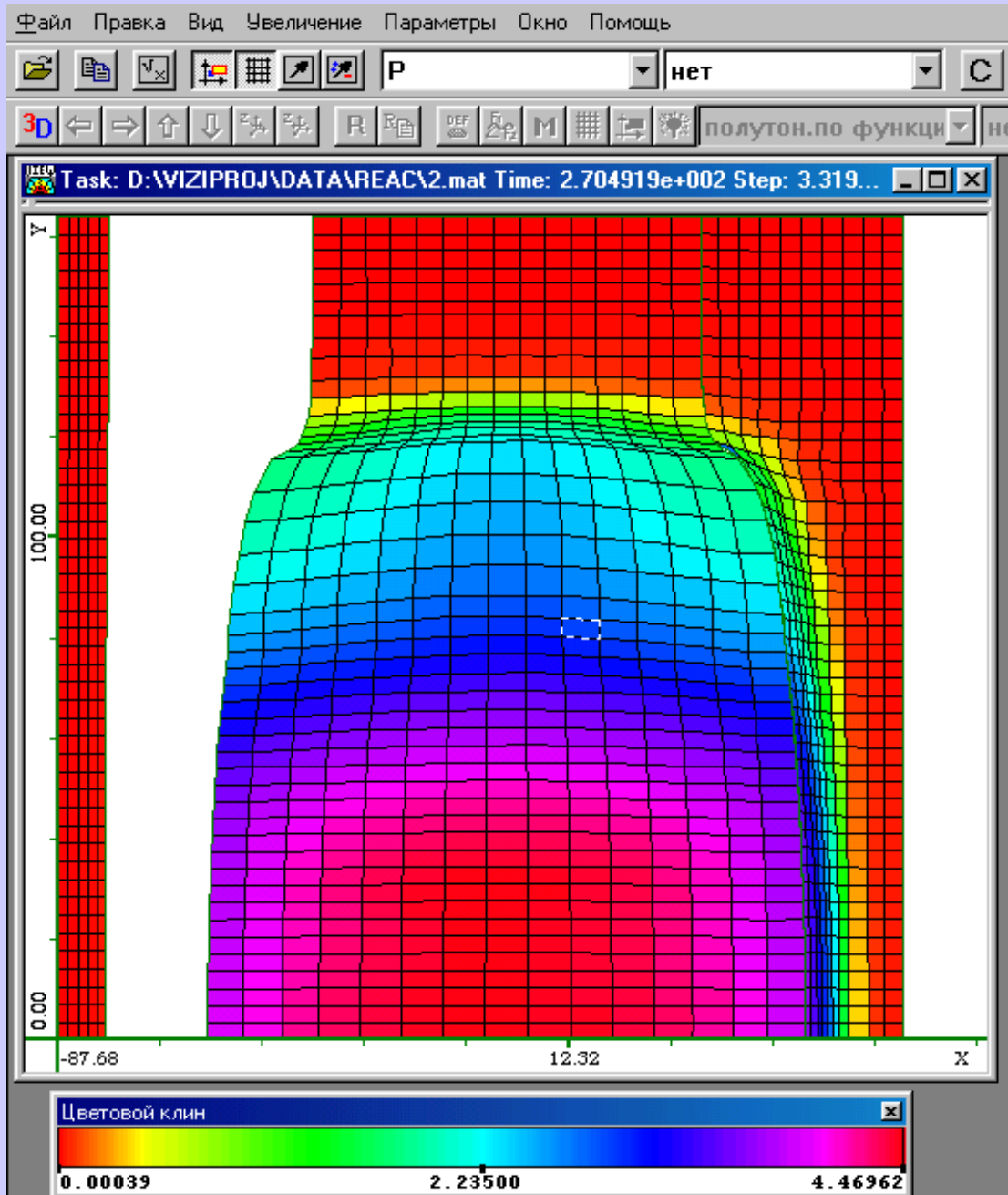


Fig. 3. The interface of system of visualization “VIZI”. Distribution of pressure at regular difference grid is presented. Linear Law of Correspondence (LLC).

2. Data Preprocessing

The determination of a minimum and a maximum values of physical characteristics over the whole grid of model: P_{\max} и P_{\min} .

3. Linear Law of Correspondence (LLC) between the Cells Color and a Physical Characteristics in the Cells

3.1 The algorithm description

Considering the linear law of correspondence (LLC) is important methodologically for further development of the subject and the comparative analysis.

The main advantage of the given algorithm is its simplicity of implementation.

Frequently presents good results.

The task lies in associating the continuous segment $I_p = [P_{\max}, P_{\min}]$ with the discrete integer-valued segment $I_c = [0, MC]$.

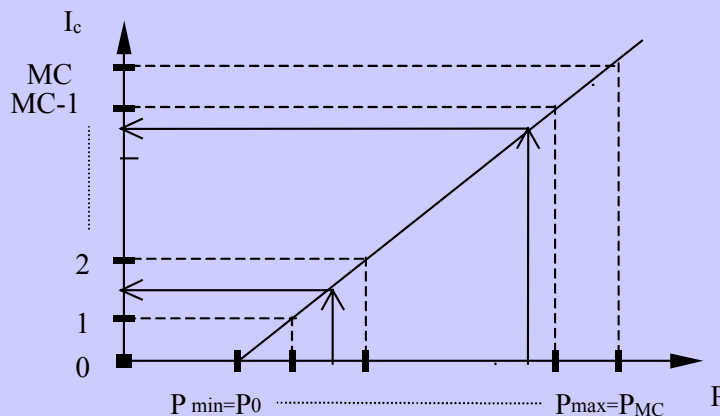


Fig. 4. LLC

Two variants of calculating a grid of color number in a palette for a cell over P value in this cell.

The first variant:

Coefficient of proportionality k :

$$k = (P_{\max} - P_{\min}) / MC.$$

Color index:

$$I_{ij} = (\text{int})(P_{ij} / k) + 0.5).$$

The second variant:

Equation of a straight line for the correspondence law:

$$(x - 0) / (P_{\max} - P_{\min}) = y / MC \Rightarrow y = (x * MC) / (P_{\max} - P_{\min}).$$

In the result obtain the first variant but in the other way when substituting:

$$y = I_{ij}, x = P_{ij}$$

3.1 Examples of the linear law work, its imperfection and peculiarities

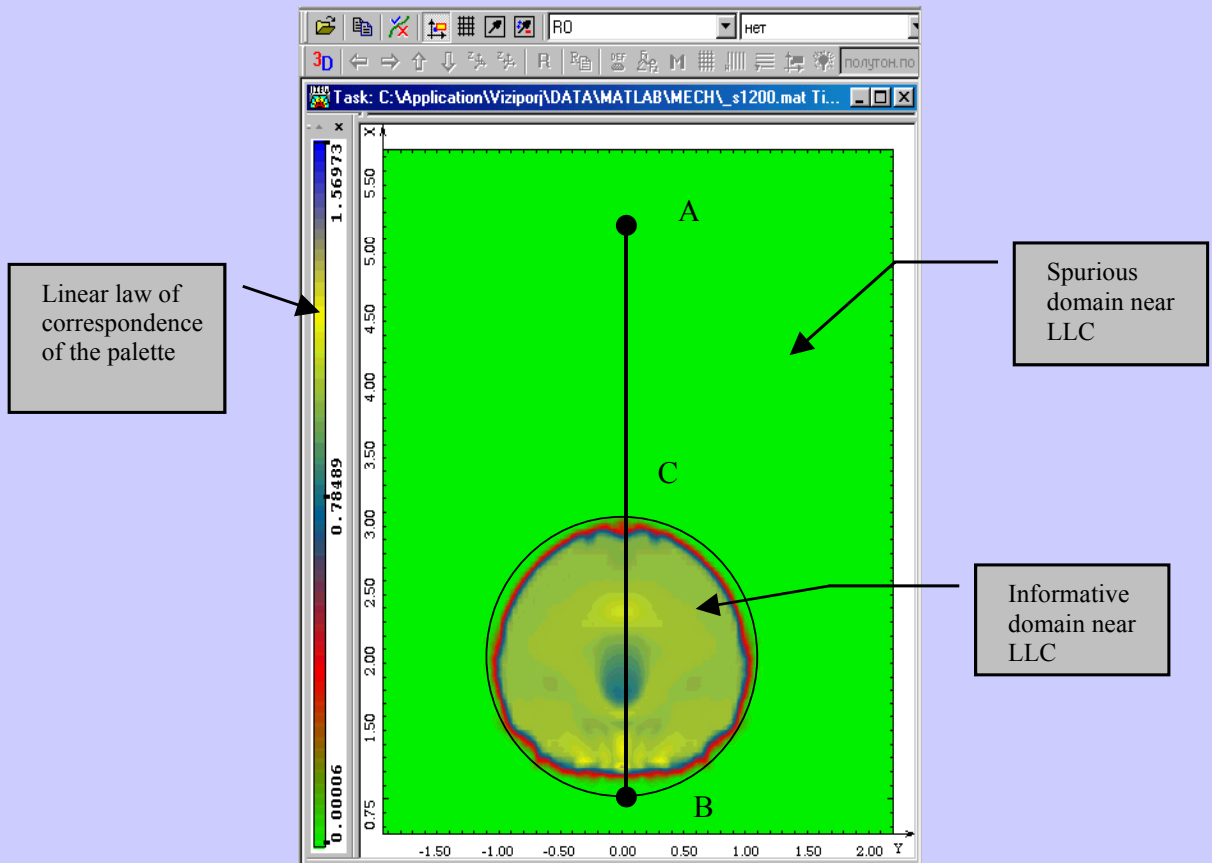


Fig. 5. We consider the LLC peculiarities. The image results to be ill-informative.

We construct the object section along AB line in Figure 5 and get the 1D graph of P value dependence on some line at the definitional domain (Fig. 6a and 6b).

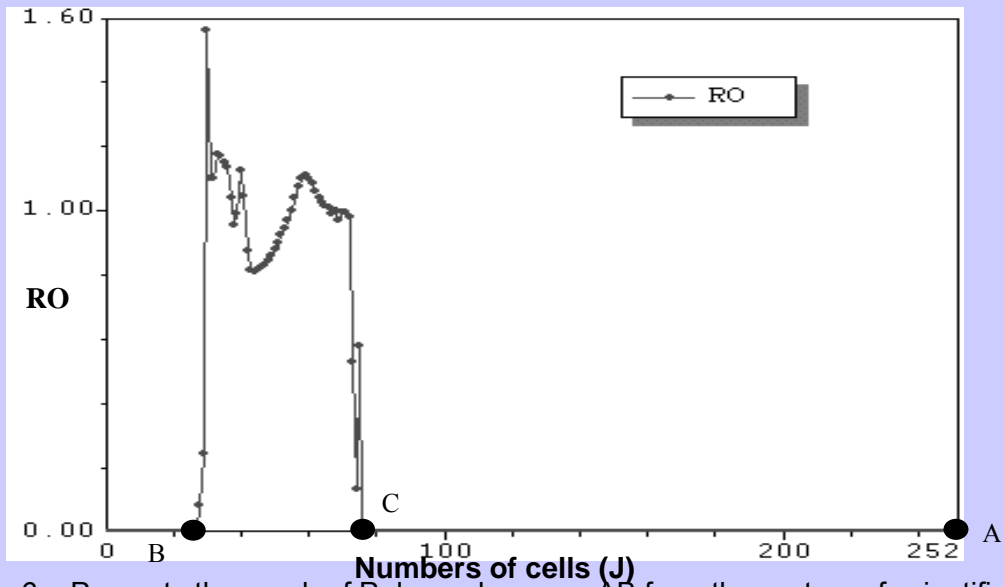


Fig. 6a. Presents the graph of P dependence on AB from the system of scientific visualization VIZI.

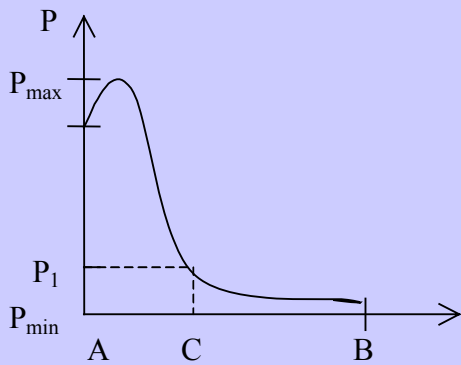


Fig. 6b.

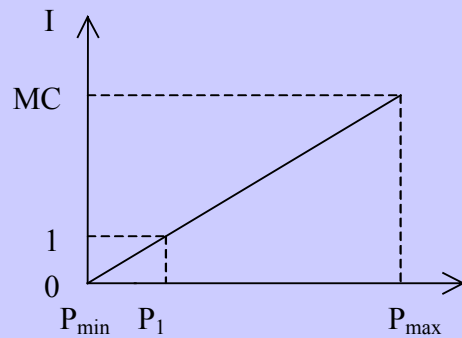


Fig. 6c.

In Figures 6b and 6c the graph 6a and the graph of LLC are presented schematically.

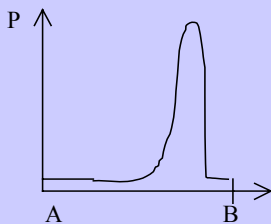


Fig. 7a.

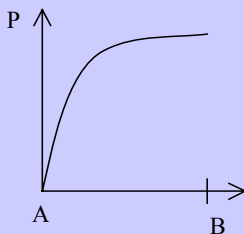


Fig. 7b.

Similar cases.

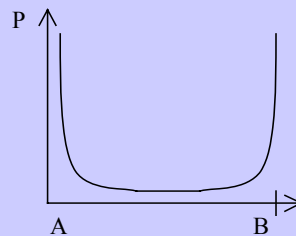


Fig. 7c.

LLC algorithm works well for data with smoothly varying P characteristics on the grid. Fig.3.

LLC is equivalent to decomposition of the segment I_p into MC well-ordered intervals, each of these having the color index.

4. Statistical nonlinear law of correspondence (SLC) between color of cells and a physical characteristic in cells.

4.1 General information

Statistical nonlinear law of correspondence (SLC) works well in cases similar to the ones described in Figures 5, 6 and 7.

Concept includes statistics gathering, which correlate quantitative information with qualitative one, i. e. with P value.

On the basis of the gathered statistics we find the colors for the cells.

This algorithm is automatic, recommended for the described cases.

At first we describe the three algorithms for statistics gathering, from this point on the algorithm of the creation of a law of correspondence works identically.

It is necessary to scan all the grid cells and fill in the statistics array STAT [MC], using LLC. At the initial initialization all the values of the array STAT equal to 0.

4.2 The first algorithms for statistics gathering (Quantitative)

A rule for filling the cells. We consider the current cell and calculate according to LLC a color index $I_{Pi,j}$.

We change a value within STAT array in a cell with the number $I_{Pi,j}$ in the following way: $STAT [I_{Pi,j}] = STAT [I_{Pi,j}] + 1$.

We perform statistics gathering over the whole grid, i. e. we calculate a number of cells, which correspond to each color.

In the result we get the 1D dependence (Fig. 8).

Then we find a maximum and a minimum values within STAT array. Correspondingly \max_s and \min_s are these values. The first algorithm of statistics gathering is finished.

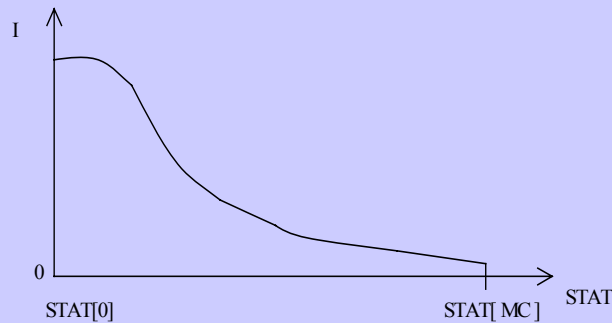


Fig. 8.

4.3 The second algorithm of statistics gathering (Areal)

In this algorithm the cells areas will be the statistics criterion. We divide explicitly I_p into MC intervals, i. e. we find the dividing points.

Within the array REG[MC] we write down the coordinates of the dividing points: $REG[I] = P_{\min} + I * (P_{\max} - P_{\min}) / MC = P_{\min} + I * k$, where $I \in [0, MC]$

A rule for filling the statistics array STAT. We consider the current cell of the grid (i, j). We find a number of the interval I_p , into which a value of $P_{i,j}$ characteristics of the current cell falls.

If $REG[I] \leq P_{i,j} \leq REG[I+1]$, then $P_{i,j}$ belongs to an interval with number I, where $I \in [0, MC-1]$.

Then we calculate the cell area according to Guron formula:

$$SQR_{i,j} = 1/2 * ((x_{i,j} - x_{i+1,j}) * (y_{i,j} - y_{i+1,j}) + (x_{i+1,j} - x_{i+1,j+1}) * (y_{i+1,j} - y_{i+1,j+1}) + (x_{i+1,j+1} - x_{i,j+1}) * (y_{i+1,j+1} - y_{i,j+1}) + (x_{i,j+1} - x_{i,j}) * (y_{i,j+1} - y_{i,j}));$$

We fill in STAT array: $STAT[I_{P_{i,j}}] = STAT[I_{P_{i,j}}] + SQR_{i,j}$.

We find the total area of the grid cells (SUM_SQR):

$$SUM_SQR = SUM_SQR + SQR_{i,j}.$$

We perform the normalization of STAT array in the following way:

$$STAT[I] = STAT[I] / SUM_SQR;$$

Statistics gathering is finished.

4.4 The third algorithm of statistics gathering (Divisions of the areas)

This algorithm of statistics gathering also uses the information about cells area but it is remarkable for its finer processing. In this case we need P values in the grid points not in cells.

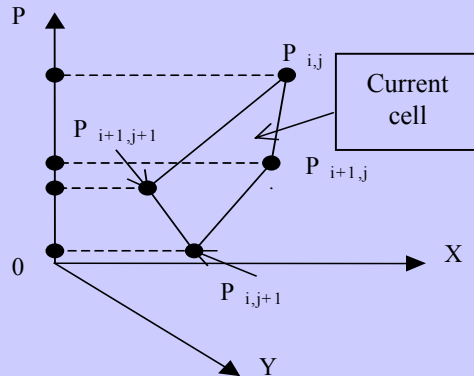


Fig. 9.

A rule for statistics gathering. We consider the current cell.

We calculate the total area of all the cells SUM_SQR .

Using LLC we find a number of the interval, in which the point characteristics of P lie.

In the result we get for the four points of the cell

$P_p = \{ P_{ij}, P_{i+1,j}, P_{i,j+1}, P_{i+1,j+1} \}$ the four numbers of color

$l_p = \{ l_{ij}, l_{i+1,j}, l_{i,j+1}, l_{i+1,j+1} \}$.

In the general case we obtain that the segment $l_0^p = [\max P_{y_3}, \min P_{y_3}]$ passes several intervals and the segment $l_0^c = [\max l_{y_3}, \min l_{y_3}]$ passes through several colors in the palette.

We calculate how many intervals NUM the segment l_0^p covers:

$NUM = \max l_{y_3} - \min l_{y_3} + 1$;

Then we divide the cell area uniformly by all the intervals with numbers $l \in [\max l_{y_3}, \min l_{y_3}]$.

We fill in STAT array: $STAT [l] = SQR_{ij} / NUM$.

It is possible to consider other ways of dividing the cells areas by the intervals (SQR), for example, the non-uniform dividing of the area by the intervals.

We perform the normalization of STAT array in the following way:

$STAT [l] = STAT [l] / SUM_SQR$;

Statistics gathering is finished.

4.5 Constructing the piecewise-linear law of correspondence for statistical data

At this step we have STAT array with the statistical information gathered above.

At the abscissa axis we decompose the segment I_p into intervals, and at the axis of ordinates we plot off the values of STAT array for corresponding intervals. Fig. 10.

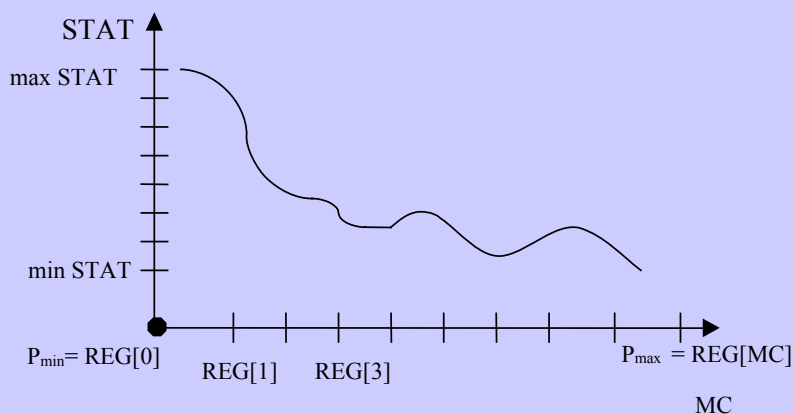


Fig. 10.

Values in STAT array are to be normalized to the range from 0 to MC, i. e. to bring in correspondence with a number of colors in the palette.

Let $\max \text{STAT}[J]$ and $\min \text{STAT}[J]$ are correspondingly the maximum and the minimum values in STAT array.

The normalization is performed according to the following formula:
$$\text{STAT}[I] = (\text{STAT}[I] - \min \text{STAT}[J]) / ((\max \text{STAT}[J] - \min \text{STAT}[J]) / MC);$$

In the result of all the actions we obtained the information, with which one may construct piecewise-linear, quadratic or cubic splines or interpolating polynomial.

We consider the process of constructing a piecewise-linear spline. In Figure 11 the points mark STAT values for each interval, joining them with segments we obtain a continuous curve, i. e. a spline of the first order.

The very obtained curve will be the nonlinear statistical law of correspondence between color and a physical characteristics of P. The straight line in Figure 11 shows the corresponding LLC from p.3.

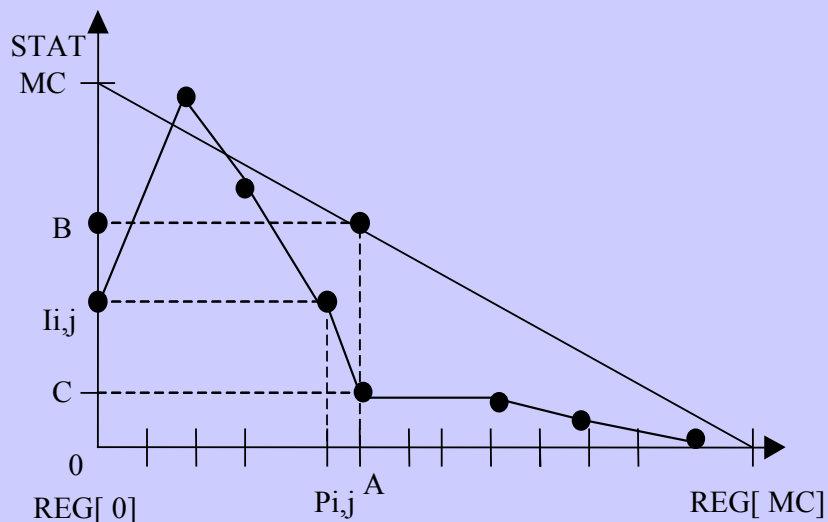


Fig. 11.

For calculating a number of colors in the palette for the cells using the formulas of the spline of the first order.

At SLC the value P_{ij} are to be substitute into the formula of the spline:

$$I_{ij} = STAT [I] * ((REG[I + 1] - P_{ij}) / k) + STAT [I + 1] * ((P_{ij} - REG [I]) / k).$$

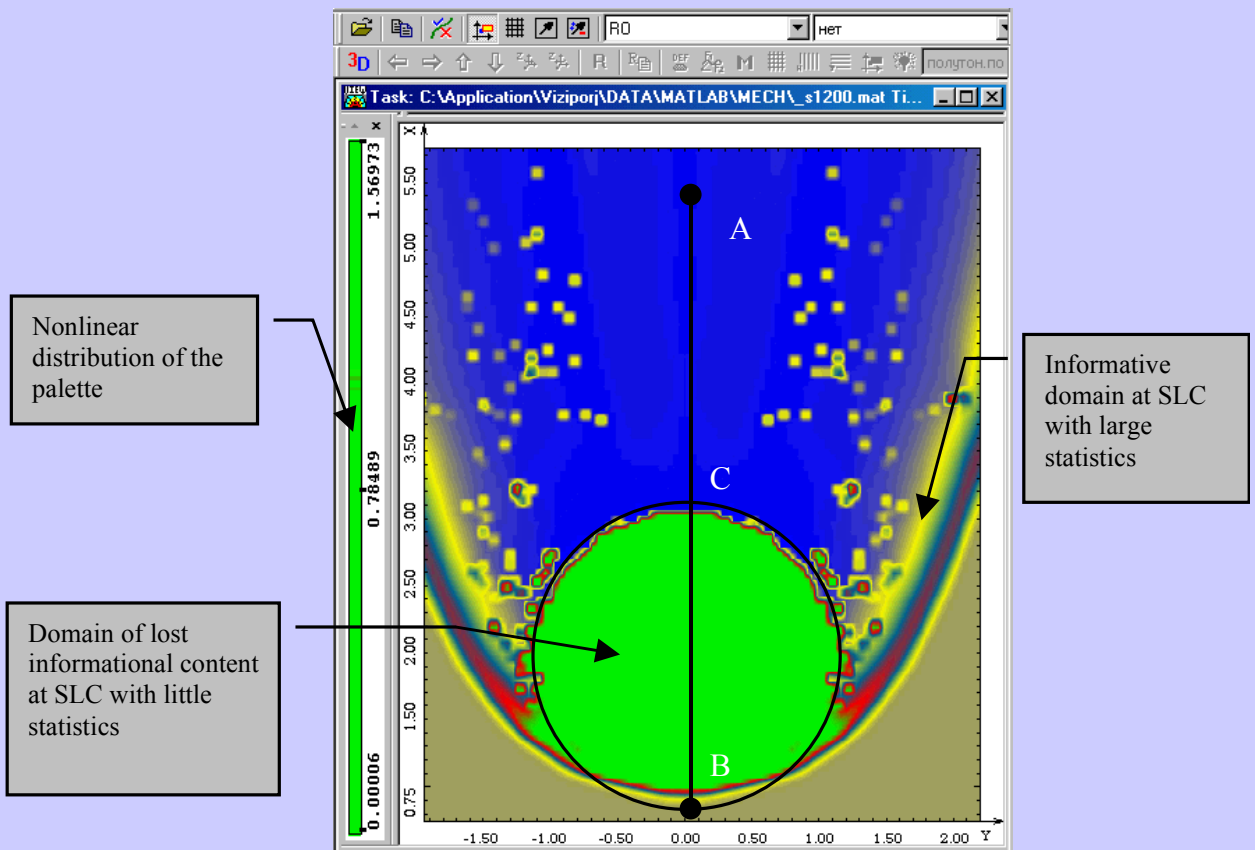


Fig. 12. Presents an example of SLC work for the same model as in Figure 5.

4.6 Methods of SLC evaluating

Note 1: Applying some other methods of statistical information gathering. We give several examples without analyzing them:

- One may consider not a scalar value P , but its gradient or its change on the grid;
- One may combine the results of statistics gathered by different methods.

Note 2: For various data the individual fitting of a method of statistics gathering is required.

Note 3: After statistics gathering one may apply different interpolating methods, piecewise-linear, quadratic or cubic splines and interpolating polynomials.

Note 4: In 4.4 we used the uniform dividing the cell area into a number of intervals, which cover the segment $[\max P_{ij}, \min P_{ij}]$. One may apply non-uniform more precise dividing of the area for statistics gathering.

Note 5: Generally speaking, P value is a function of two-variables, therefore one may propose statistics gathering by some set of 1D model sections.

Main feature of SLC: Statistics gathering allow reducing the multidimensional distribution of P scalar value to 1D dependence of the palette colors at color interpretation of P value. The good properties are:

- The way of how to improve the image information content becomes clearer;
- The palette is also a 1D array that allows presenting the correspondence between P and the palette visually and simply;
- There is a possibility to apply 1D splines and interpolating polynomials.

4.7 Imperfections

1. Some additional time and memory resources are required for statistics gathering.
2. Informational content for the image disappears for the range P with little statistics. Below in 4.8 we give a method for eliminating this imperfection. (Fig. 12)

The given SLC gives a bad result in cases when P value is distributed over the grid in some measure uniformly, without big sudden changes or when the grid is of very small dimensionality. Figure 13a presents an example of such an image. Figures 13b and 13c present the sketchy graphs of typical sections and Figure 13d presents the graph of such real section for the model in Figure 13a.

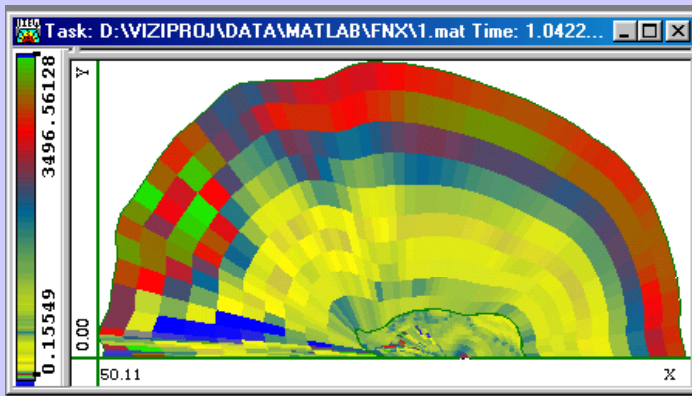


Fig. 13a.

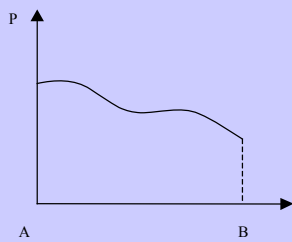


Fig. 13b.

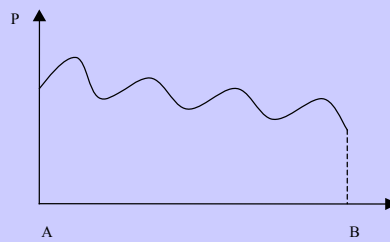


Fig. 13c.

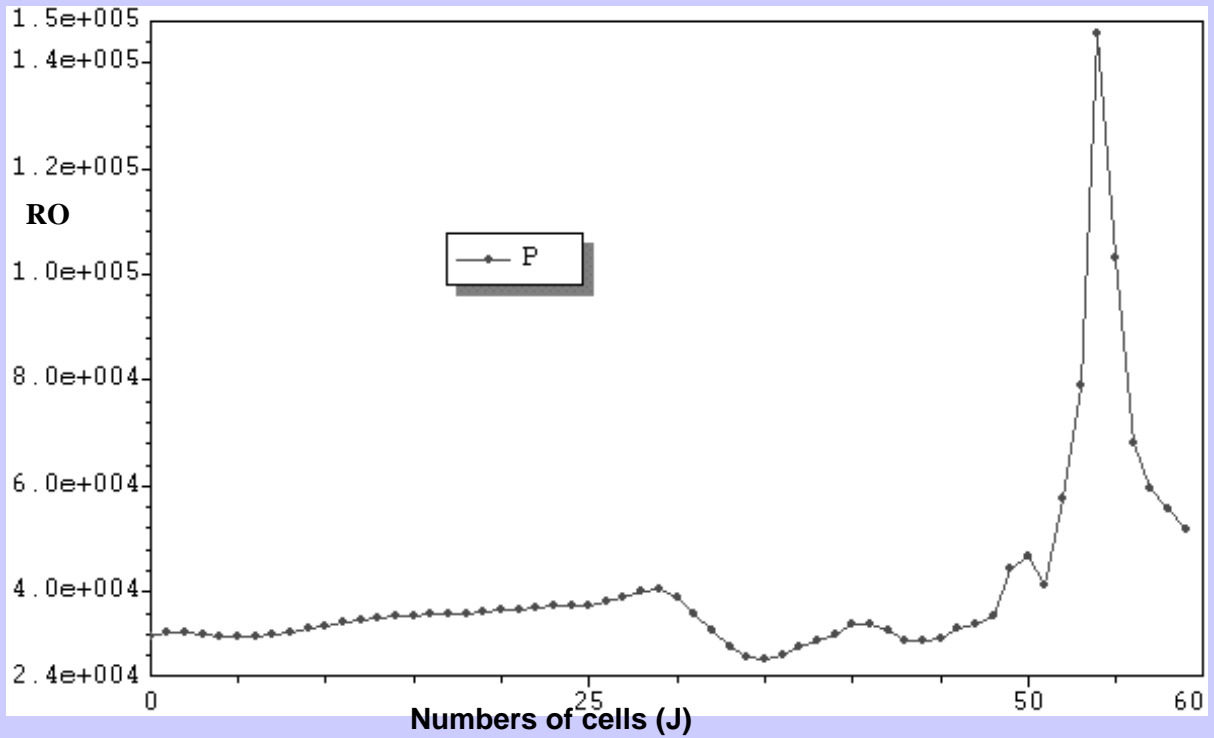


Fig. 13d.

In the process of statistics gathering there occur dips or peaks in some intervals, i. e. there is no statistics in them because of lack of information or because it is unreasonably large. (Fig.14).

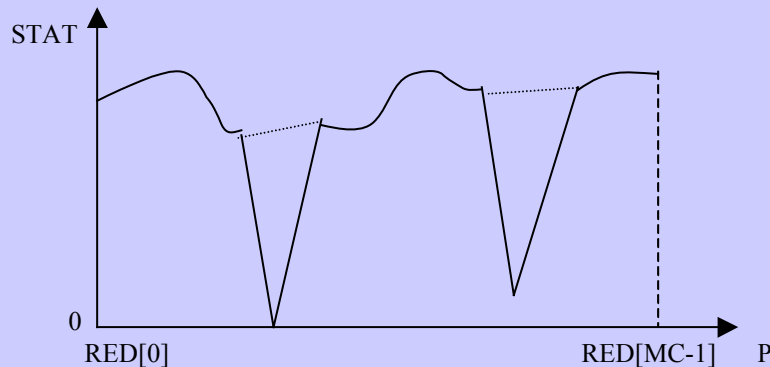


Fig. 14.

The reason of this lies small changes of P value and/or great discreteness of the grid. By the algorithm of statistics gathering the dips are also included into the resulting spline, that leads to abrupt changes of colors and this does not confirm with the real situation. This imperfection may be eliminated in the different ways:

The first method: To use different laws of correspondence, for example, LLC.

The second method: To use a different algorithm for statistics gathering.

The third method: Statistics correction implies eliminating of dips or “noises”. It means the elimination of non-informative intervals of the segment I_p decomposition or their integration. (Fig.14).

4. Colors confusion. It means that different P values may correspond to one color in the palette. It takes place in places where the derivative of SLC graph changes its sign.

The comments are as follows:

- Color interpretation refers to the qualitative data analysis and to the trending, therefore together with the indices in informational messages this imperfection is not of great significance;

5. Algorithm for SLC correction with considering LLC

This algorithm (SLC+LLC) allows eliminating the mutual imperfection of linear and statistical laws of correspondence.

There occurs a concept of integrating these two algorithms.

Below we give the two methods for merging these laws.

Method 1: One should determine the intervals for the segment I_p , in which we will apply the linear law and in which — the nonlinear law. This method has one weak point, i. e. fitting the points of a law changing. It should be noted that such a point may not be a single one. The comments are as follows:

- One may consider algorithms, which automatically fit these points. But all the criteria for finding the points of a law changing would have partly an empirical or approximate character.
- One may apply a method of manual points fitting, it is a visual-empirical method. It that an engineer, while looking at the statistics graph of P or P of conditional 1D argument, puts or enters interactively the dividing points of the segment I_p .

Method 2 (SLC+LLC): The concept of this method lies in automatic joining the two laws of correspondence. Assume that statistics has been gathered by some method, i. e. we have the normalized array STAT. We consider the current cell. We calculate for it a number of color by LLC: I_n . Further we calculate for it a number of color by SLC: I_{kn} . We form the resulting color number I_{pes} :

$$I_{pes} = (I_n + I_{kn}) / 2;$$

Some other ways of joining the color numbers of I_n and I_{kn} are also possible.

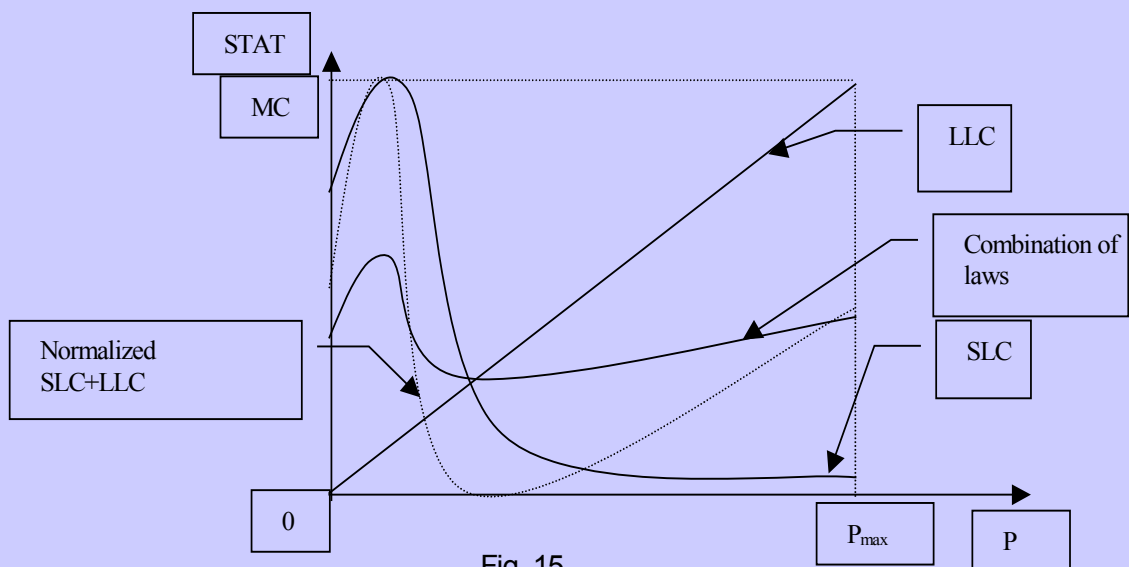


Fig. 15.

The given method gives excellent results in practice. (Fig. 18)

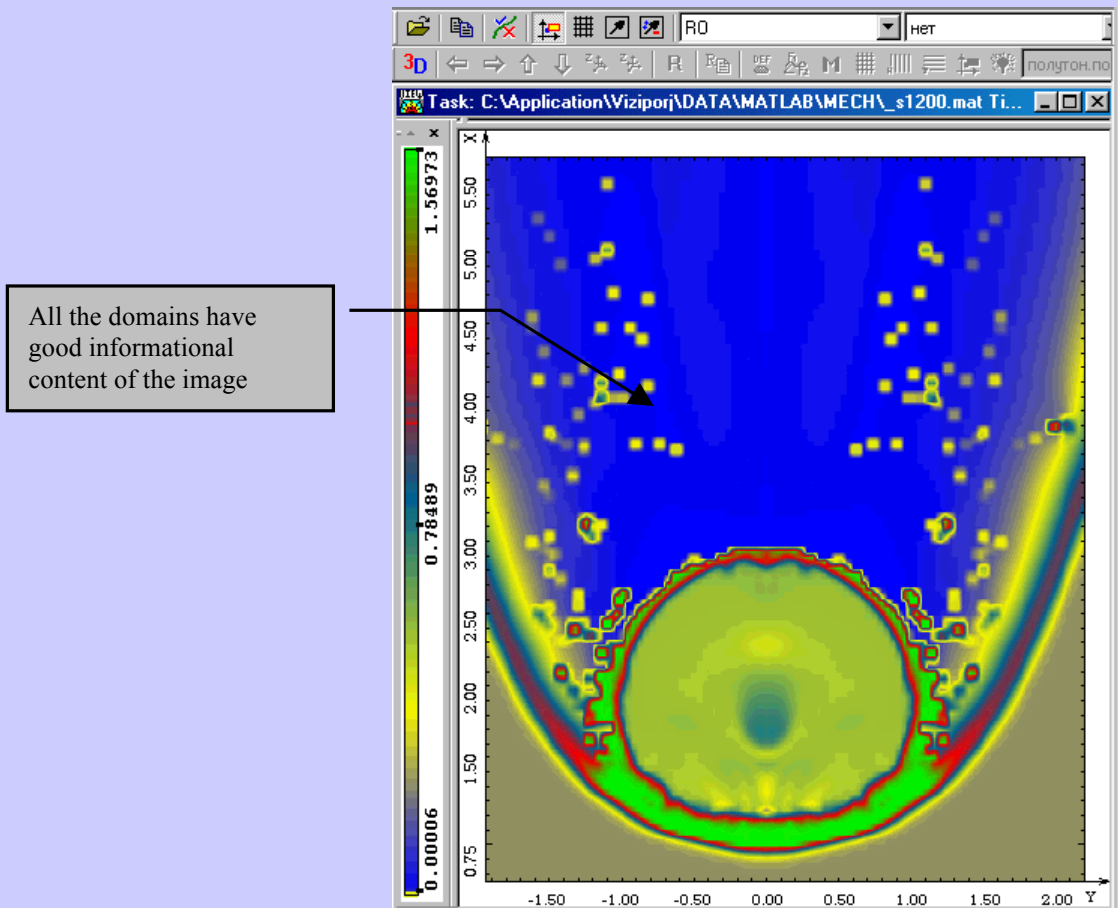


Fig. 16.

6. Other methods of forming nonlinear laws of correspondence between color and physical characteristics

This part of report is briefly deals with two algorithms for forming nonlinear laws, which are sometimes used in graphical systems.

6.1 Algorithm for forming nonlinear exponential law of correspondence

This algorithm combines automaticity and interactivity of constructing the nonlinear laws of correspondence. Its concept lies in applying standard analytical functions or their algebraic combinations.

There is an infinite set of these functions, so it is natural that one should fit its well-founded set and determine variable parameters in analytical formulas for their interactive changing.

Without preprocessing one typically can not apply analytical expressions, since each specific function has its own definitional domain, i. e. a strictly defined geometrical place for potential arguments, a specific curvature shape of a graph.

We consider the peculiarity of this method at the example of the exponential function: $f(x) = e^x$.

We consider its graph and, for example, the object section profile. In order to get the color of a specific cell we come up to the formula: $I_{ij} = (e^{\alpha P_{ij}} - 1) / ((e^{\alpha P_{max}} - 1) / MC)$.

The example of work of exponential law and the reverse exponential law are given in Figures 17-20.

To change the result one may apply the several ways:

- Changing α parameter;
- Parallel transfer of the function graph along the abscissa axis;
- Applying the law of the reverse exponent;
- Applying the combination of the exponential law and the law of the reverse exponent.

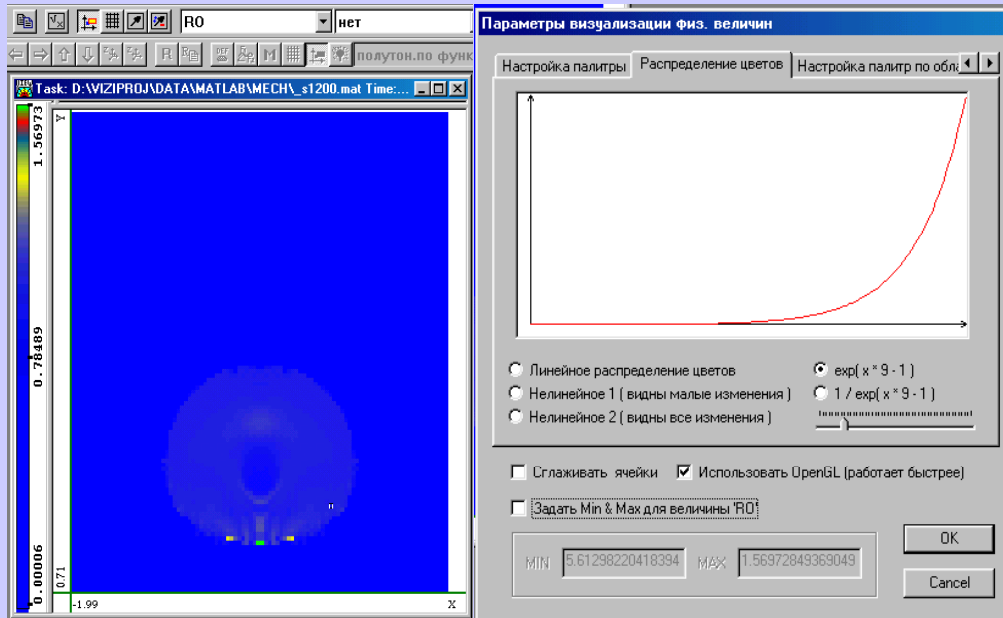


Fig. 17. Shows that the exponential law at $\alpha = 9$ gives a bad result over the whole image.

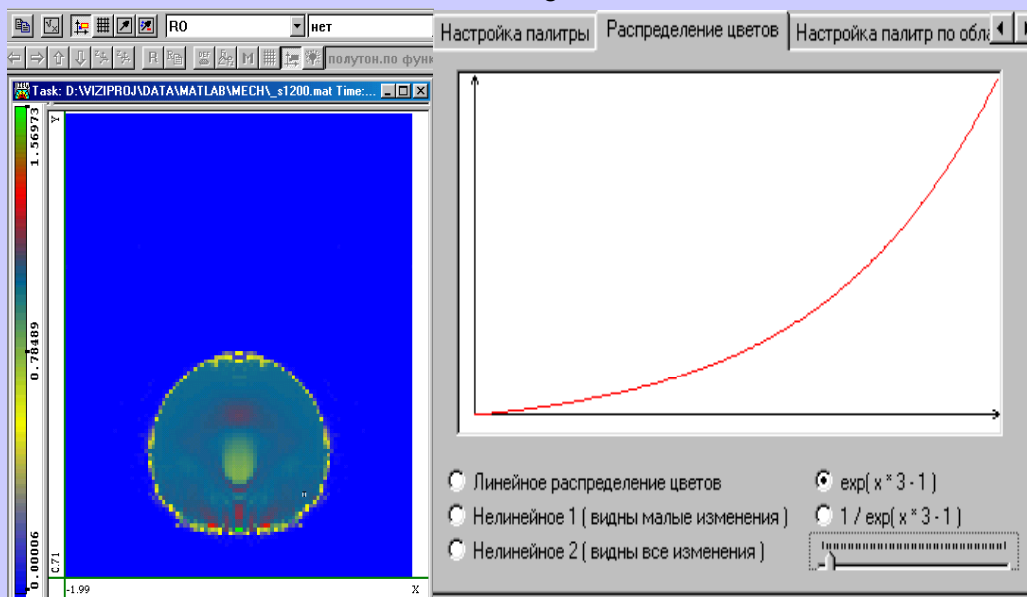


Fig. 18. Shows that the exponential law at $\alpha = 3$ gives a good result at great changes of a characteristics.

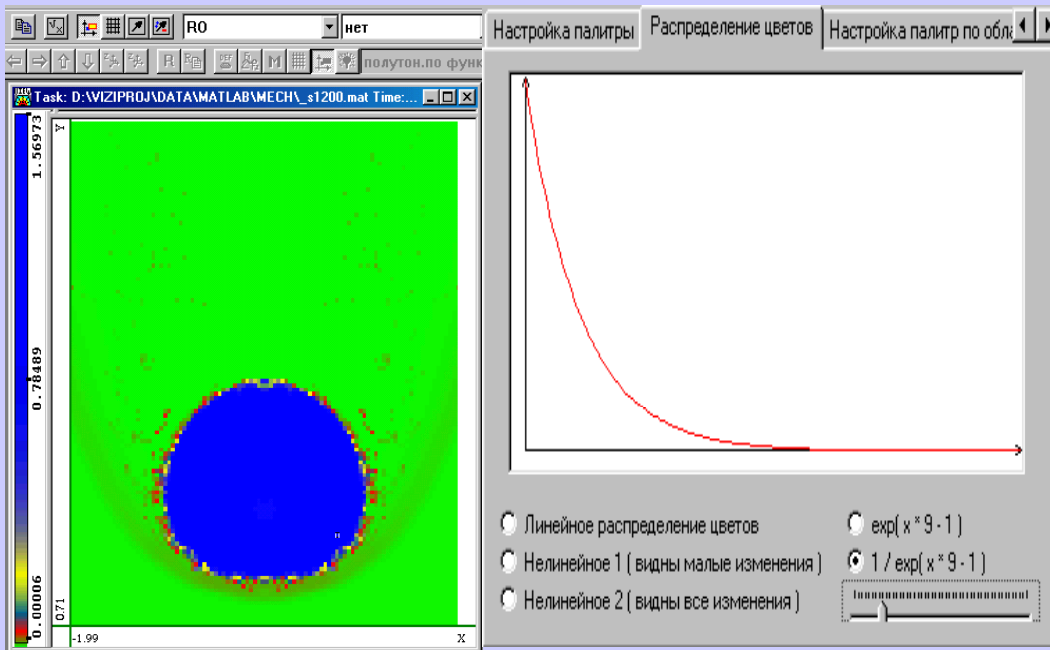


Fig. 19. Show that the law of the reverse exponent at $\alpha = 9$ give good results at small changes of a characteristics.

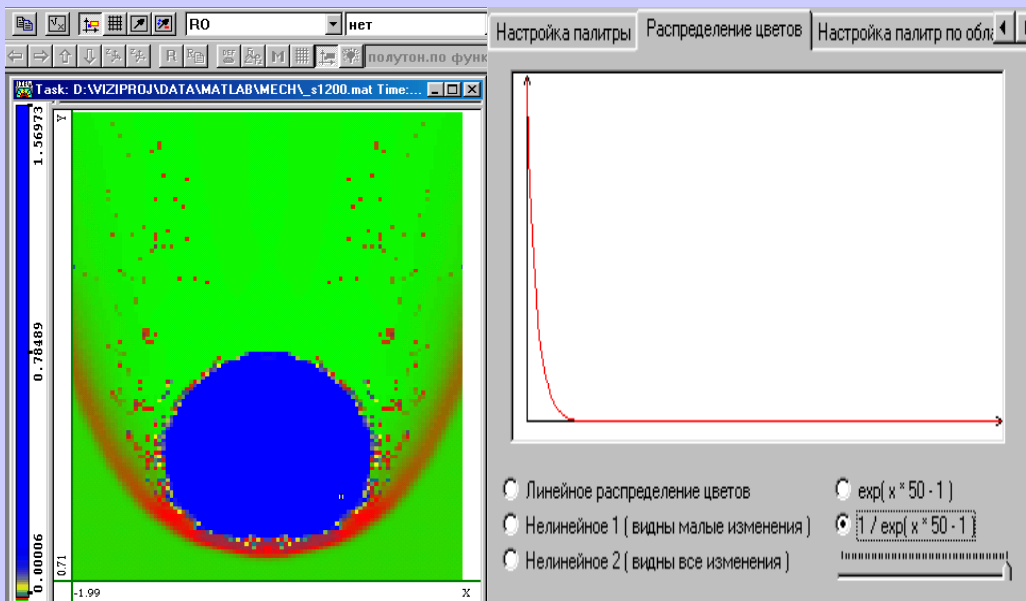


Fig. 20. Show that the law of the reverse exponent at $\alpha = 50$ give good results at small changes of a characteristics.

6.2 Algorithm of manual fitting of piecewise-linear law of correspondence.

This method is frequently used in graphical systems and packages.

It is necessary that a statistics graph or a graph of a typical section of the object would present nearby. Fig. 22.

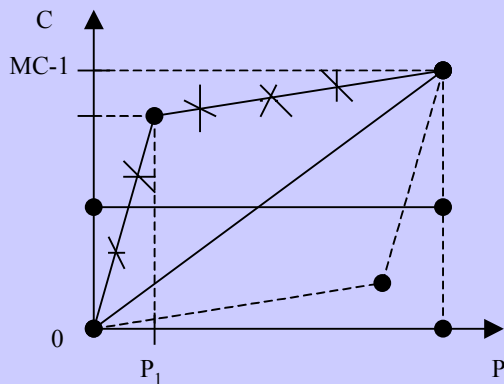


Fig. 21.

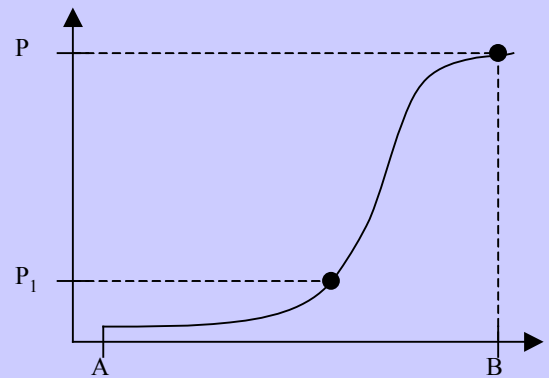


Fig. 22.

7. Conclusion.

In the conclusion we note that statistical laws and the law of the reverse exponent with parameters are successfully implemented in the system of scientific visualization "VIZI2D". All the illustrations from VIZI were presented above at one mathematical model for better comparative analysis. Below in Figures 23 – 26 the results for another model are presented.

System of scientific visualization "VIZI2D" was created by members of "scientific visualization" laboratory of RFNC-VNIITF in 1996. Now the functional meaning of the system and structural modification is taken place.

The give system was presented at various conferences.

Functional meaning of "VIZI" system is oriented to visualization of results of 2D numerical simulation of mathematical physics problems. The system works with data presented with regular difference grids.

The main result: Considerable improvement of informational content of objects at graphical interpretation of results of mathematical simulation of physical processes.

Summarizing: In the given paper the algorithms were stated at the example of 2D regular grid, however these algorithms may be applied without changes for some other data structures, for example:

- 3D regular grids;
- Irregular grids;
- Gridless models.

Further investigations of nonlinear laws of correspondence continue along the following trends:

- Other methods of statistics gathering;
- Other principles of laws forming, for example, a palette fitting;
- A detailed investigation of analytical laws;
- Methods of estimation of an object informational content or measures of information amount within an image.

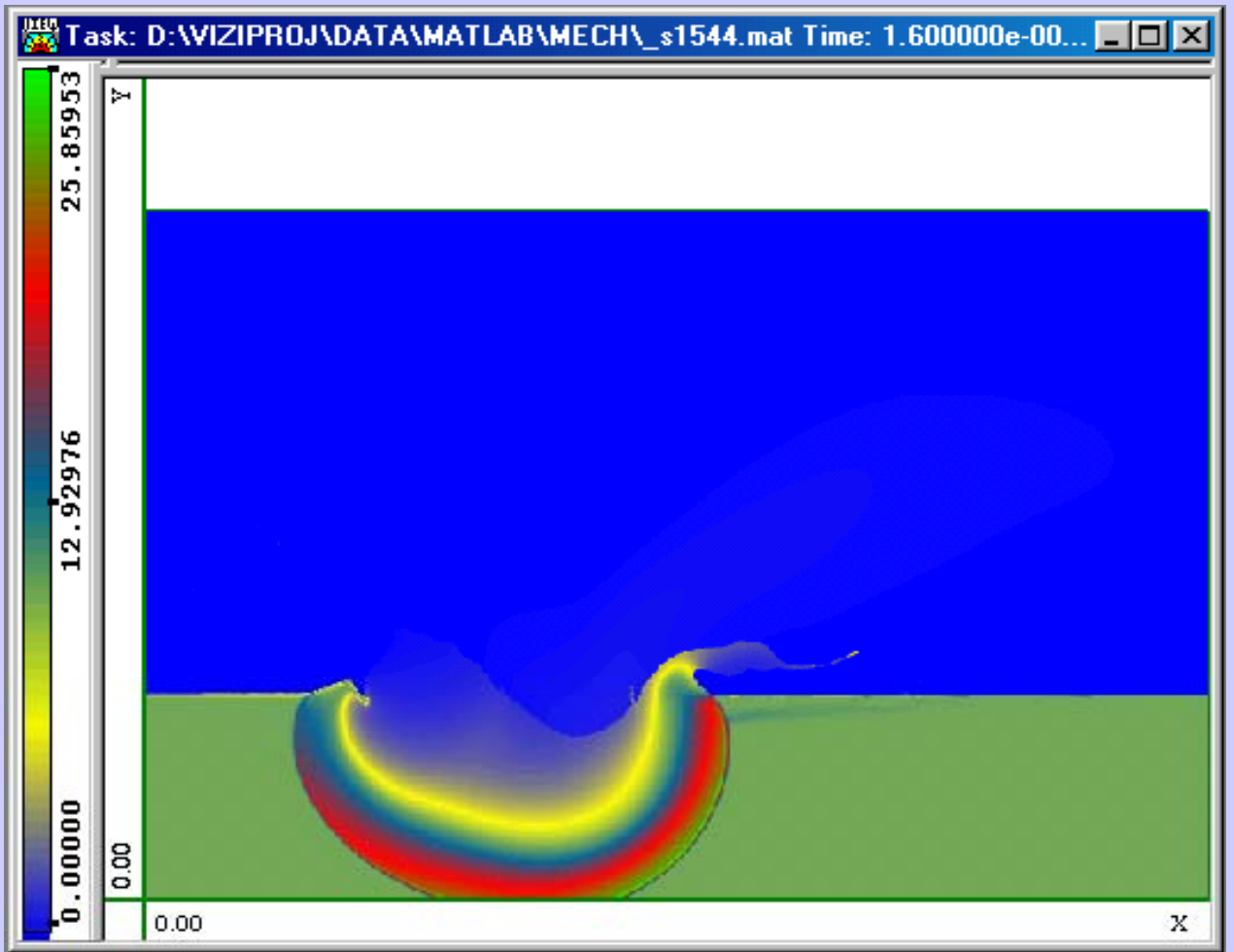


Fig. 23. (LLC)

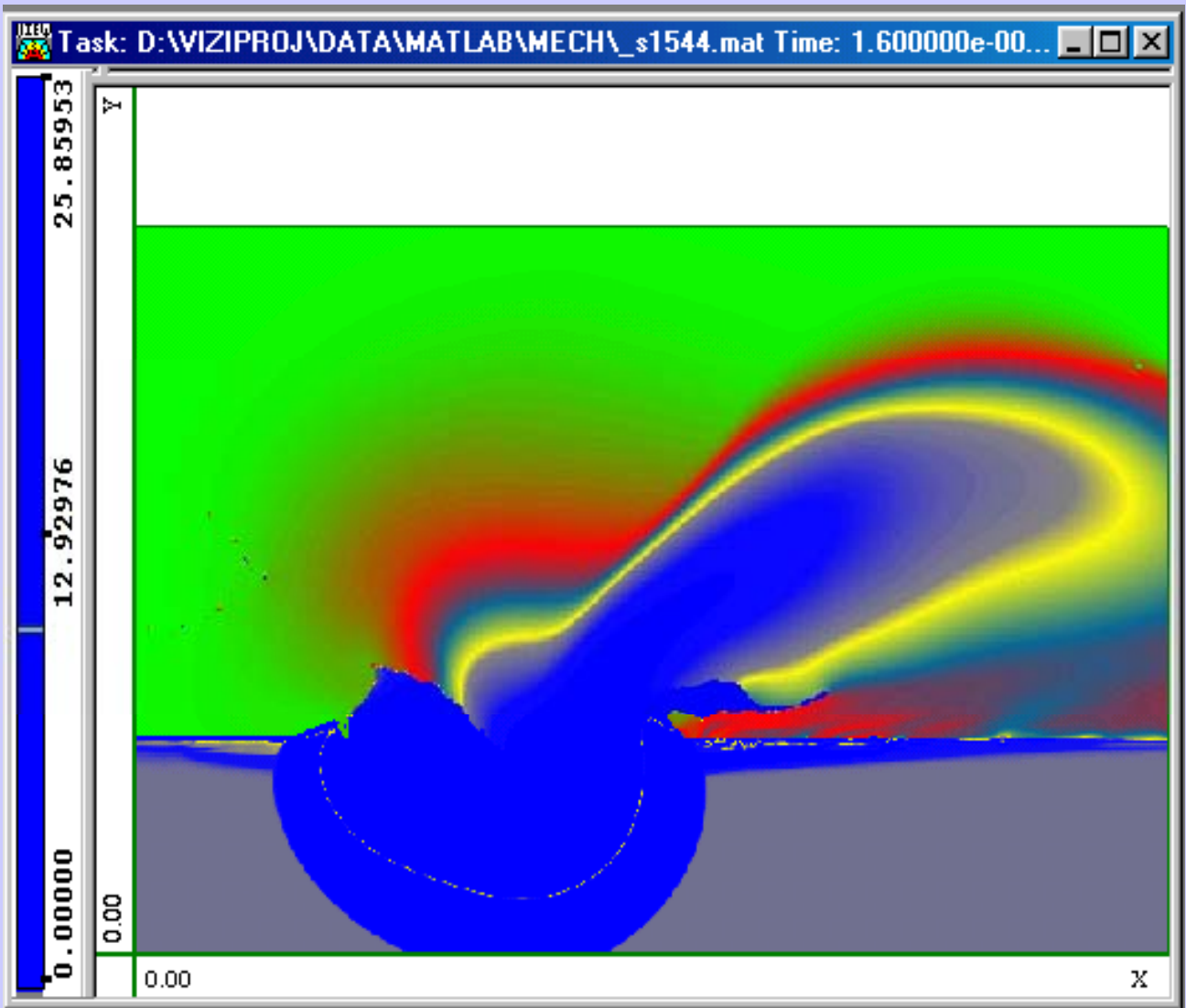


Fig. 24. (SLC)

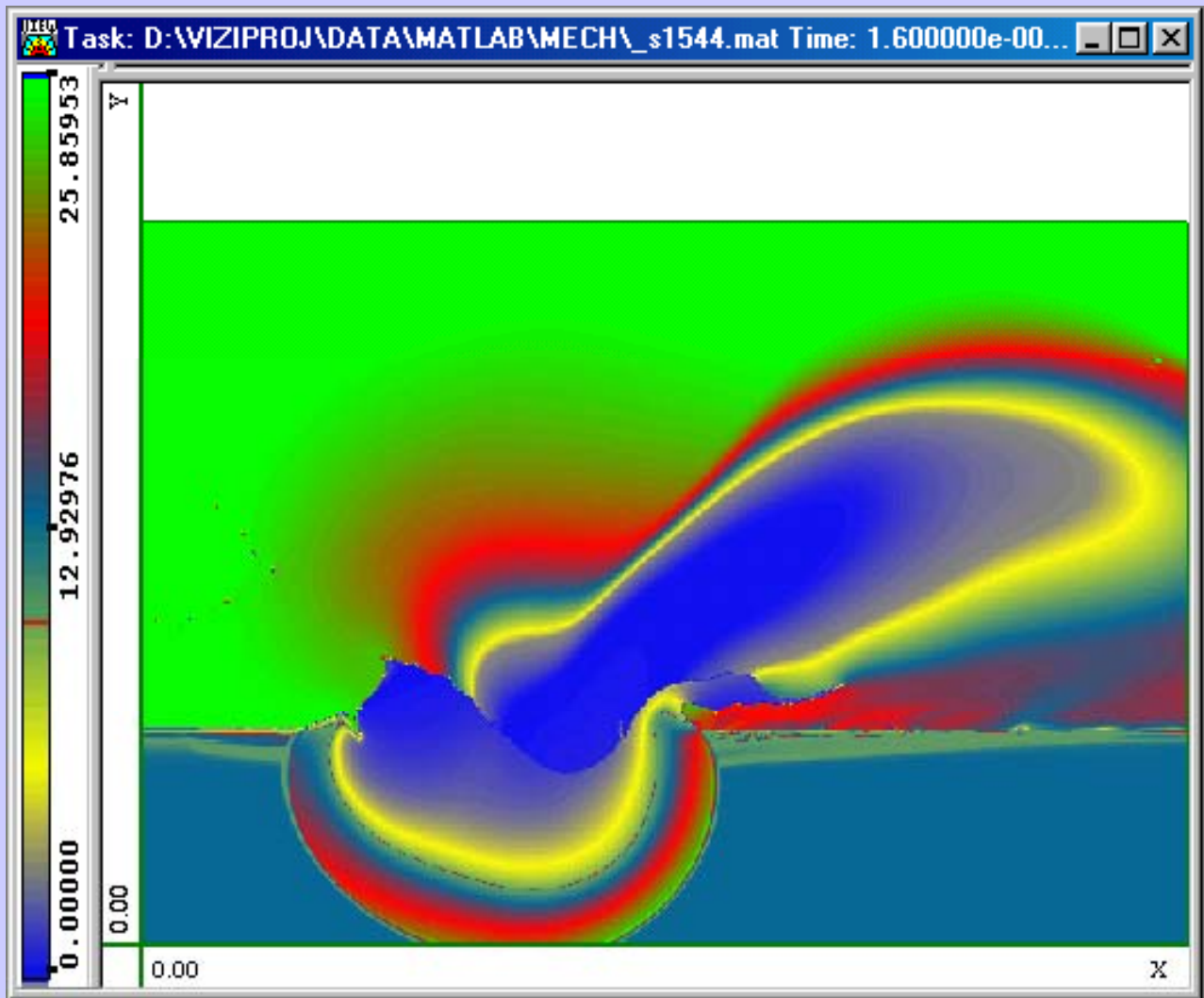


Fig. 25 (SLC+LLC)

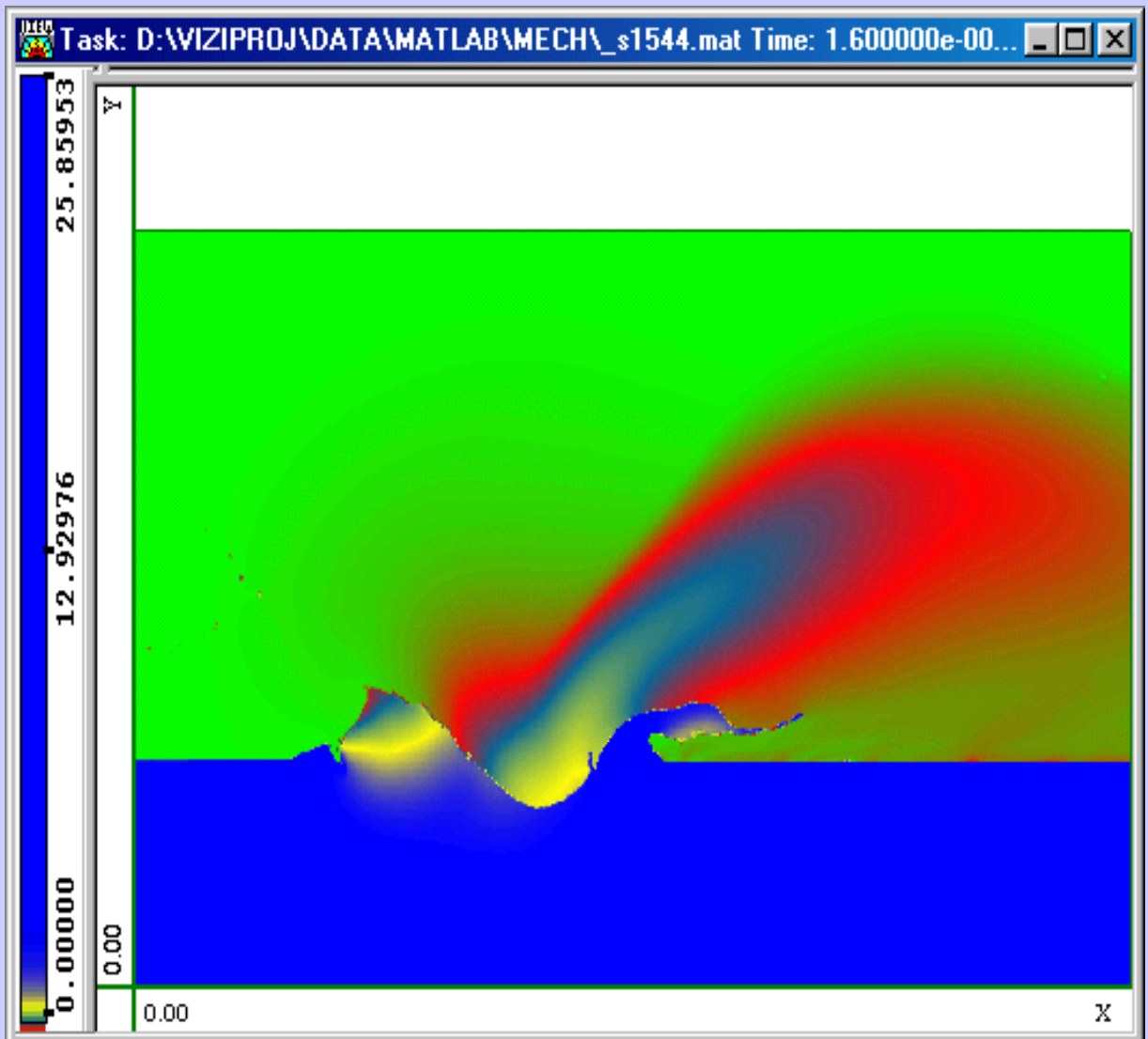


Fig. 26 (the law of the reverse exponent)