AMLESAC: A New Maximum Likelihood Robust Estimator

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Abstract

Robust parameter estimation methods are the general tool in computer vision, widely used for such tasks as multiple view relation estimation and camera calibration. In this paper, we propose a new general robust maximum-likelihood estimator called AMLESAC, which is a noise adaptive variant of renowned MLESAC estimator. It adopts the same sampling strategy and seeks the solution to maximize the likelihood rather then some heuristic measure, but unlike MLESAC, it simultaneously estimates the outlier share γ and inlier noise level σ. Effective optimization for computation speed-up is also introduced. Results are given for both synthetic and real test data for different types of models. The algorithm is demonstrated to outperform previous approaches for the task of pose estimation and provide results equal or superior to other robust estimators in other tests.

Keywords: Robust Estimation, Sampling Consensus, Maximum Likelihood, RANSAC, MLESAC, Pose Estimation, Non-Linear Optimization

1. INTRODUCTION

One of the key tasks in the field of computer vision is to establish a relation between information extracted from images and some mathematical parametric model. The general task of parameter estimation from measured data can be formulated in the following way. We need to estimate the parameters vector θ, so that

$$F(X,\theta)=0; \quad (1)$$

where $X=\{x_i\}, i=1..N$ – is the measured data vector, $F(X,\theta)$ is the target mathematical model. A classical example of parameter estimation task is fitting straight line to a set of points. In this case, x is the point set, $F(x,\theta)$ is line equation, θ - line parameters to be estimated.

Statistical parameter estimation methods such as maximization of likelihood (ML) or maximization of posterior probability (MAP) rely on certain assumptions on relation of measured data and target model. It is expected that all data $x=\{x_i\}$ is generated by the target model $F(x,\theta)$, and later deviated by noise ε, with expectation $M(e)=0$, and variance $D(e)=\sigma^2, \sigma=const$. This assumption does not hold in cases when part of the measured data is generated not by the estimated model, but originate from some pure measurement noise. The data points generated by the target model are called *inliers* with respect to the model F, generated by some other model (i.e. noise) are called *outliers*. Outliers do not conform to the target model, and the result of parameter estimation in presence of outliers by likelihood maximization may deviate arbitrarily far from the true values [10]. The proportion of inliers in the input data is usually denoted by $\gamma \in [0,1]$. An estimation technique is called robust if it can estimate the model from input data with inlier fraction $\gamma < 1$.

Several methods were proposed to deal with presence of outliers – M-Estimators [10], voting schemes like Hough transform [2],[10], Least Median of Squares [3],[10], or a family of methods based on RANdom SAmpling Consensus estimator (RANSAC) [1].

2. BACKGROUND

RANdom SAmpling Consensus (RANSAC) was proposed by Fisher and Boules in 1981 [1]. It uses the following strategy. The measured data has total of N samples with unknown fraction of inliers γ. To estimate true model parameters we would like to label data as outliers and inliers and estimate the model parameters from inliers only. As this labeling is initially unknown, RANSAC tries to find outlier-free data subset randomly, in several attempts. To maximize the probability of selecting sample without outliers RANSAC tests only samples of minimal size.

The RANSAC algorithm consists of M iteration of the following three steps:

1) Random sampling m elements of the input data $S_k \subset x$
2) Estimating hypothesis θ_k from S_k
3) Measuring the hypothesis score $R_k = R(\theta_k)$

After generation and evaluation of M hypothesizes, the one with highest score $R=\max\limits_{k=1,N}(R_k)$, is selected as the result of robust estimation.

Given the expected fraction of inliers γ in the input data and the total number of samples N, the number of algorithm iterations M necessary to find the true model parameters with desired probability P can be calculated [10].

Since the introduction of the original RANSAC estimator, several different approaches for hypothesis scoring were developed. The original RANSAC scheme counts the number of inliers for the each of the generated hypothesis and selects the one that maximizes this number:

$$R(\theta)=\sum_i p(r_i(\theta)^2),\; p(r_i^2)=\begin{cases}0 & r_i^2<T^2\\ 1 & r_i^2>T^2\end{cases}, i=\overline{1,N} \quad (2)$$

where $r_i(\theta)^2$ is discrepancy of point x_i and hypothesis θ, T – outlier rejection threshold.

The MSAC estimator [4] raises the influence of quality of each particular inlier by measuring the quality of hypothesis as

$$R(\theta) = \sum_i p(r_i(\theta)^2),\ p(r_i^2) = \begin{cases} r_i^2 & r_i^2 \le T^2 \\ T^2 & r_i^2 > T^2 \end{cases}, i = \overline{1,N} \quad (3)$$

MLESAC [5] extends robust estimation to evaluate true hypothesis likelihood instead of heuristic measures (2), (3). This requires estimation of inlier share γ, which is solved by iterative EM algorithm [5].

The above-mentioned methods make no us of the prior knowledge that can be very helpful for more accurate hypothesizes evaluation. The MAximum aPosteriory Samling Consensus (MAPSAC) extends MLESAC estimator by replacing the likelihood with posterior probability. However, in most cases, the absence of meaningful prior information reduces it back to likelihood.

One of the main drawbacks of previously mentioned methods is relying on parameters estimated elsewhere - outlier threshold T or predefined noise parameter σ^2 and inlier share γ (except for MLESAC that estimates γ). If their values are far from true, the scoring is incorrect and result can deviate arbitrary far from true model. Unfortunately, this case is not uncommon as will be discussed in next section in more detail.

Simultaneously with RANSAC another robust estimator was proposed from mathematical community. It uses median error for hypothesis evaluation, to lower outlier's contribution:

$$R(\theta) = median(r_i(\theta)^2), i = \overline{1,N} \quad (4)$$

It does not rely on pre-defined thresholds, but was demonstrated to show lower performance than that of MSAC and its other methods in such computer vision tasks as two-view relation estimation [4].

For fundamental matrix robust estimation a new method was proposed by Feng and Hung in [12]. It is based on MSAC scoring function, but to increase the precision of fundamental matrix estimation it extends the EM algorithm from MLESAC to both inlier fraction γ and noise deviation σ estimation. However, our test has shown that accuracy of σ and γ is very poor in many cases and cannot be reliably used without modifications for real test sequences. Our estimator uses likelihood for hypothesis scoring instead MSAC heuristic scoring function and simultaneously accurately estimates σ and γ using local optimization and subset of input data selection.

3. THE VARIANCE OF NOISE IN SEQUENCE OF ESTIMATIONS

General robust estimators require predefined parameters of noise distribution. Portion of inliers γ and noise standard deviation σ are used to calculated the outlier threshold T in RANSAC and MSAC, and directly used for hypothesizes likelihood and posteriori probability calculation in MLESAC and MAPSAC. In many cases these parameters can be manually selected or be estimated by the input data generation method, like point feature detector [4]. But often this is not possible due to natural variations of error parameters. One of such examples, which actually stimulated development of general noise adaptive robust estimator, is given below.

3.1 Pose estimation problem

Consider set of 2d points projected from a set of 3d points. The set of 3d points $D = \{d_i\}, i = 1,...,N$ is projected to corresponding points $M = \{m_i\}, i = 1,...,N$ in 2d image by projection matrix P:

$$m_i = P(d_i), i = 1,...,N$$

The camera orientation and position that is encoded in P are called camera pose. So the task of estimation of P from known sets D and M is called the pose estimation problem.

One pose estimation method is 6-point linear method [11], which works in both Euclidian and Projective frameworks. It solves the pose estimation problem by directly solving a system of linear equations $m_i = P(d_i), i = 1,...,N$. The quality of estimated pose P' is measured as a sum of reprojection errors of all 3d points:

$$R(P') = \sum_i dist(m_i, P'(d_i))^2 ,$$

where $dist(.,.)$ is distance between two points in image.

3.2 The variance of noise

Consider camera pose estimation for the frame k when the pose for frames $\{1,...,k-1\}$ is already estimated. In camera pose estimation point feature detection is used to estimate $M = \{m_i\}, i = 1,...,N$. The detector estimates the coordinates of m_i with certain error ε_{match} that is almost constant between frames. This gives the first source of error. The 3d points $D = \{d_i\}, i = 1,...,N$ are calculated by triangulation [11] from point feature tracks in several frames with known poses P . Since both poses and feature tracks were estimated with errors, the resulting 3d points D coordinates were also calculated with some error, which parameters are extremely complicated to estimate. Also it is not constant and varies from frame to frame. This is the second source of error.

Since pose estimation error depends on both sources of error, one of them unknown and varying, it makes impossible to select fixed outlier threshold T or inlier share γ and deviation σ for all frames in image sequence. This leads to camera calibration failures in wide number of image sequences if general RANSAC-based estimators are used.

Similar situation often arise in other computer vision tasks. If input data is extracted directly from the images, the error of point measurement is almost constant for all frames. However, if part of input data is the result of previous estimations the deviation of noise σ and inlier fraction γ are can vary from frame to frame. In such cases these parameters should be estimated for each hypothesis scoring.

4. PROPOSED METHOD

For accurate robust estimation of model parameters we propose a novel algorithm called AMLESAC. It is based on general random sampling consensus framework but introduces maximum likelihood estimation of hypothesis with simultaneous noise parameters estimation. We also propose local optimization and subset selection to increase precision and speed of estimation.

4.1 Maximum likelihood error

We assume that inlier point coordinates are measured with error that satisfies Gaussian distribution with zero mean and standard deviation σ . The probability distribution for error of inlier points $x_i \in \Re^s, x_i \in X$ with s dimensions is given by:

$$p_{inlier}(e_i \mid \sigma) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^s \exp\left(-\frac{e_i^2}{2\sigma^2}\right)$$

where $e_i = \left\|\tilde{\hat{x}}_i - x_i\right\|$ is L2 norm of difference between measured x_i and maximum likelihood estimate $\tilde{\hat{x}}_i$ of true point $\bar{x}_i$, $\tilde{\hat{x}}_i$ is a point that satisfies the relation $F(\tilde{x}_i, \theta) = 0$ and minimizes the L2 norm $e_i = \left\|\tilde{\hat{x}}_i - x_i\right\|$.

The distribution of outliers is assumed to be uniform (the least informative distribution), with s dimensions and given by:

$$p_{outlier}(e_i) = \frac{1}{v}$$

where v is volume of space within which the outliers are believed to fall uniformly.

If no prior information about points being inlier or outlier is given, the probability of point to be inlier is equal for each point from input data. The error for all points can be modeled as mixture of Gaussian and uniform distributions:

$$p\ (e_i \mid \sigma) = \gamma \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^s \exp\left(-\frac{e_i^2}{2\sigma^2}\right) + (1-\gamma)\frac{1}{v}$$

The probability of generating data D with respect to parameter hypothesis θ is given by:

$$P(D \mid \theta) = \prod_i^N \left(\gamma \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^s \exp\left(-\frac{e_i^2}{2\sigma^2}\right) + (1-\gamma)\frac{1}{v}\right),$$

where N is number of data points. The negative log likelihood –L of all points equals:

$$-L = -\sum_i^N \log\left(\gamma \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^s \exp\left(-\frac{e_i^2}{2\sigma^2}\right) + (1-\gamma)\frac{1}{v}\right)$$

The true hypothesis θ minimizes the negative log likelihood. The search for such hypotheses is dubbed “maximum likelihood estimation”. We select negative log likelihood –L as a scoring function

4.2 Mixture model parameters estimation

The value of hypothesis likelihood depends on γ and σ and can be estimated correctly only is the values are accurate. As it was discussed in details in section 3 these parameters can usually vary for different input data sets and should be estimated for each hypothesis.

We propose the method for mixture parameter estimation based on median of point errors for all input data points. Consider for the one-dimensional case the minimum inlier fraction $\gamma_{\min}$ and number of iteration steps n_{iter}, we test all variants of $\gamma \in [\gamma_{\min}, 1]$ with certain step γ_{step}

1) Form a vector of discrepancy errors for all input data points $E = \{e_i\}, i = 1, N$ where $e_i = \left\|\tilde{x}_i - x_i\right\|$
2) Sort E so that $e_i < e_j, \forall i, j : i < j$
3) For each $\gamma_k \in [\gamma_{\min}, 1]$ with step γ_{step}
   a. Select $E_k = \{e_i\}, i = 1, (\gamma_k * N)$
   b. Set $\sigma_k = 1.4826 * median(E_k^{\ 2})$
   c. Estimate γ_k using EM algorithm from MLESAC [5]
   d. Calculate negative log likelihood $-L_k$
4) Select γ , σ that correspond to maximum $-L_k$
5) Then σ is refined by using gradient descent minimization method applied to negative log likelihood

Another iterative method for inlier fraction γ and inlier noise deviation σ estimation was proposed by Feng and Hung in [12]. Their method is an extension of EM algorithm for inlier fraction γ estimation only, which was originally proposed by Torr and Zisserman in [5]

4.3 Stopping criteria

Considering the inlier fraction γ in input data and number of RANSAC iterations M the probability of selection the sample with size p during this iterations equals:

$$\Gamma(\gamma, M) = 1 - (1 - \gamma^p)^M$$

Two different stopping criteria exist for RANSAC-based estimators to achieve the desired probability of outlier-free sample selection. First is to compute the number of iterations M before estimation using the prior information about inlier fraction γ in input data, and stop the estimator only after all M hypotheses are selected and tested. Conservative strategy is often used in this case and inlier fraction γ is assumed lower than expected.

The second strategy is to compute $\Gamma(\gamma, k)$ after each iteration. The inlier share γ for currently best hypothesis is used. $\Gamma(\gamma, k)$ is the probability that outlier-free sample has already been selected after k iterations. If $\Gamma(\gamma, k)$ is higher than a given value (usually 95-97%), then RANSAC estimator stops

The last criterion is not applicable in case when noise deviation σ and inlier fraction γ are estimated for each hypothesis separately as AMLESAC does. It is not uncommon that a large noise deviation σ and relatively large inlier fraction γ are estimated for erroneous hypothesis from sample with outliers. The probability $\Gamma(\gamma,k)$ is large in this case and algorithm can stop long before outlier free sample can actually be found. So only the first criterion can be used in AMLESAC. This was not considered in [12] where both σ and γ were also estimated for each hypothesis.

4.4 Mixture parameter estimation on subset of input data

Estimation of σ and γ requires computation of hypothesis likelihood several times with different values of σ and γ for each hypothesis. Since the size of input data is large and counts hundred and thousands of points, the hypothesis evaluation step is usually more expensive and time consuming than sampling and hypothesis estimation steps. As a result, AMLESAC is several times slower then other RANSAC-based estimators.

In [7] and [8] two different methods for lowering the computational complexity of RANSAC-based estimators were proposed. They share the same idea of using only subset of input data for hypothesis evaluation. In [7] each hypothesis is tested against several randomly selected points, and only if specified number of them are marked as inliers the evaluation is continued using all input data. Preemptive RANSAC, introduced in [8] first estimates all M hypothesizes. The one point is randomly selected and all hypothesizes are tested against it. The partial score is calculated based on it. Next point is then selected and hypotheses scores are updated. The process is continued until the time limit is reached.

We propose to use the similar idea of subset selection to increase the speed of σ and γ estimation. After new hypothesis is generated, the subset of all input data $T_k \subset x$ is randomly selected and used in mixture model parameters estimation. In case of large volume of input data, a relatively small subset T_k is sufficient for accurate σ and γ estimation. Depending on size different share of input data can be selected for such test subset. The negative log likelihood of hypothesis is then calculated using all input data and estimated values of inlier fraction γ and noise deviation σ .

4.5 Local optimization

The assumption that good estimate can be computed from any outlier-free sample is not always correct. Since the size of each sample tested by RANSAC is minimal, the influence of noise is can be significant. For most RANSAC-based estimators this leads to errors in computation of inlier fraction γ and hence the probability that outlier-free sample has already been selected. Consequently, the number of samples M tested before termination of algorithm increases. To reduce the influence of noise hypothesis can be refined by local optimizations, as it was proposed in [9]. Consider the minimal sample size is H. After new top-score hypothesis θ is found, a new sample S_{LO} is randomly selected from points that have been marked as inliers with respect to hypothesis θ. Size of S_{LO} is larger then H. The exact size of S_{LO} can be determined from used model parameter estimation method. Hypothesis θ is then re-estimated from S_{LO}. It was demonstrated that this step lowers the probability of rejecting the hypothesis generated from outlier-free sample [9].

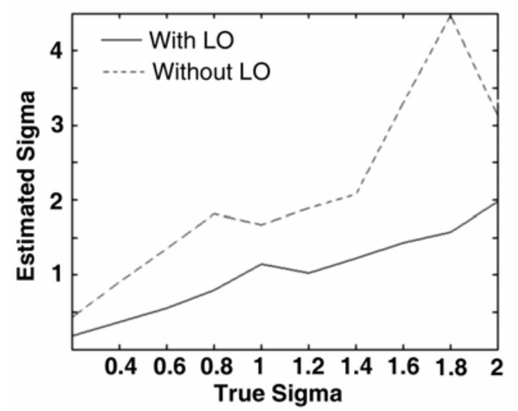


Figure 1 Inlier noise deviation σ estimation with and without local optimization

The accuracy of hypothesis scoring greatly depends on precision of σ and γ estimation. Our tests have shown that this precision is very low if hypotheses are estimated on minimal samples that makes σ and γ estimation meaningless. We propose to apply local optimization step to each hypotheses before σ and γ estimation. This significantly increases the precision of σ and γ estimation as shown on Figure 1. Because the inliers are not yet calculated when local optimization is applied, a sample S_{LO} is constructed from points x with lowest error $e = \left\| \tilde{x} - x \right\|$.

4.6 Algorithm summary

The proposed algorithm searches for the parameter vector θ with highest likelihood on input data x with assumption that input data is a mixture of inliers measured with error that obey the Gaussian distribution and uniformly distributed outliers.

1) Calculate the required number of iterations M based on prior estimate of γ
2) Repeat M times:
    a. Random sampling m elements of the input data $S_k \subset x$
    b. Estimate hypothesis θ_k from sample S_k
    c. Apply local optimization to refine θ_k
    d. Random sample of the input data $T_k \subset x$
    e. Estimate the mixture parameters σ and γ based on hypothesis θ_k and sample T_k
    f. Measure the likelihood of hypothesis θ_k using all input data x with estimated values for σ and γ

3) Select hypothesis θ with highest likelihood
4) Minimize the robust cost function as described in section 5

5. NON-LINEAR MINIMIZATION

The result of any RANSAC-based estimator is a vector parameter θ, which has been estimated using only a portion of input data, even if a local optimization is applied. The precision of estimation can be increased if all inliers or even all input data points are used for estimation. It has become a "gold standard" to apply non-linear minimization over all input data using robust hypothesis scoring function to refine the resulting model. The most common non-linear minimization methods for such tasks are Gauss-Newton and Levenberg-Marquardt [13].

5.1 Noise parameters estimation based on current hypothesis

One of the best general methods for model parameterization required for non-linear optimization is point-based parameterization introduced in [4][5]. It proposes to parameterize the model θ by minimal sample of input data points $S_k \subset x$. The non-linear minimization method varies coordinates of points in S_k instead of varying the parameters θ directly. For scoring function evaluation first the value of parameter vector θ is estimated from S_k as it is done in RANSAC-based estimators**,** then score itself is calculated from θ and x**.**

This parameterization has several advantages over model type specific parameterizations. First is that sample $S_k \subset x$, which has been used for best hypothesis θ estimation by RANSAC-based robust estimator serves as good starting point for non-linear minimization and has a strong support with high confidence. Second, it is consistent, which means that during non-linear optimization only hypothesizes that can actually arise are accounted for.

The first variant of our non-linear refinement step uses this point parameterization. Negative log likelihood –L is used as a scoring function for minimization. Inlier share γ and noise deviance σ are estimated during each scoring function evaluation. This increases the precision of refinement in cost of additional computation.

5.2 Free noise parameters estimation

Instead of implicit calculation of inlier fraction γ and noise deviance σ we also consider the inclusion of these parameters into model parameterization. In this case the search is conducted in space of both sample points and noise parameters.

6. EXPERIMENTS ON REAL DATA

Based on the proposed AMLESAC estimator, the camera calibration framework was developed and tested on image sequences of various real scenes. As was discusses previously in section 3, the camera pose estimation problem highlights the necessity of robust estimator that will be adaptive to noise level in input data. So it was important to assess the performance of the proposed method on this particular task.

Several real image sequences were captured with photo-camera Canon IXUS 500. The scenes were constructed from a set of man-made objects arranged on a top of the table. Feature-based calibration algorithm was then applied to image sequences. First, the point feature tracker with robust outlier detection via homography and fundamental matrix estimation was used to create a set of feature tracks. Second, the two frames based on image-based measure were selected and camera motion and scene structure were initialized similar to [11]. Then the pose estimation procedure was applied to each frame of the sequence.

The pose estimation procedure consists of robust estimator and a linear 6-point pose estimation method as described in section 3.1. After a robust estimator initialized camera pose the non-linear refinement procedure was applied.

The main goal of the real test was the comparison of whole camera calibration frameworks when MSAC and AMLESAC were used for pose estimation problem. The other robust estimators like LO-RANSAC, Preemptive-RANSAC, Guided RANSAC has been designed for lowering the computational complexity then that of MSAC or MLESAC with similar or slightly lower reliability and precision.

The accuracy of camera motion reconstruction was assessed by measuring the 3d point reprojection error in all the sequence. An additional comparison was made by visual inspection of reconstructed camera motion to check its consistency with the real trajectory.

The results of camera calibration by sequential pose estimation using MSAC and AMLESAC estimators for "cup" image sequence are shown in **Figure 2**. As can be clearly seen from **Figure 2** (b) 3d points from the surface of the cup, when reprojected using camera pose that was estimated by MSAC, lie far from the respective matches. The non-linear optimization step fails to refine the camera pose sufficiently to label these points correctly as inliers. Errors in camera pose estimation leads to erroneous structure and pose estimations for other frames. This results in obvious deviation of reconstructed camera motion from real trajectory and severe deformations of recovered 3d structure as seen in **Figure 2** (c).

The inlier noise σ deviation for this sequence is shown on **Figure 3**. Images 1 and 5 were used for structure and motion initialization, so poses of these cameras were estimated separately thus the inlier noise deviation for these frames was set to 0. A steady rise of inlier noise deviation is clearly visible on the plot. This is the main reason why MSAC fails to correctly estimate pose for all frames of this test sequence. In this case AMLESAC obviously shows the superior results as shown in Figure 2.

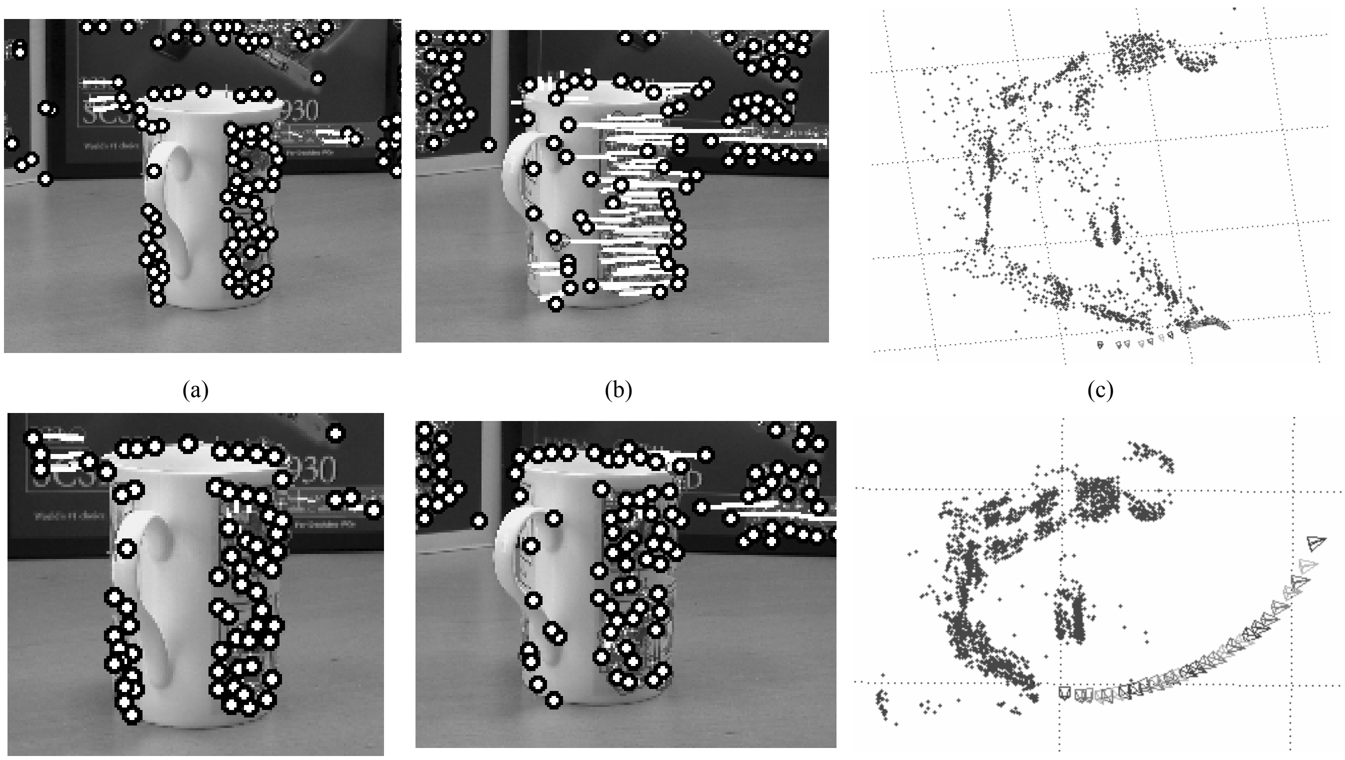


Figure 2 Camera pose estimation for “cup” image sequence (a),(b) – Reprojection of 3d points on 6-th and 9-th frames using camera poses by MSAC estimator, (c) – 3d points and camera trajectory, reconstructed using MSAC estimator. (d),(e) – Reprojection of 3d points on 6-th and 9-th frames using camera poses by AMLESAC estimator. (f) – 3d points and camera trajectory, reconstructed using AMLESAC estimator. Magnified parts of source images are shown. Reprojections are black circles with white center, reprojection error is marked by white line. Notice that MSAC estimator marks most of the points on the cup as outliers since their respective reprojection errors are large. This leads to significant errors in camera pose estimation failure in camera trajectory

On other test image sequences camera calibration framework based on AMLESAC shows a superior performance then that of based on MSAC. For “box” sequence mean reprojection error falls from 1.4 to 1.1, for “cup2” sequence it decreases from 0.92 to 0.83.

7. DISCUSSION

A number of tests on synthetic data sets were also conducted to evaluate the AMLESAC performance. The first series of tests was made on the classical problem of line fitting. The second series of tests was made for camera pose estimation problem. The tests were designed to compare the accuracy of robust estimators methods with and without simultaneous noise parameters γ and σ . The precision of γ and σ estimation is also evaluated.

7.1 Synthetic data tests – lines

First experiments were made on line fitting tasks. For each test a set of points was randomly generated on a line and perturbed with Gaussian noise with zero mean and different standard deviation σ . Then outliers were generated to uniformly fill the square region [-10,10]*[-10,10], with different inlier shares ranged in [0.1,0.8]. The first set of experiments was designed to evaluate the estimation of inlier share γ and noise deviation σ . Figure 4 (a) shows the value of estimated σ averages by 100 tests. In Figure 4 (b) the corresponding outlier share $1-\gamma$ is demonstrated. It can be seen from these figures that AMLESAC accurately estimates the inlier error deviation but tends to raise the outlier share when noise deviation increases. When noise deviation rises, more inliers fall far from the line and marked as outliers.

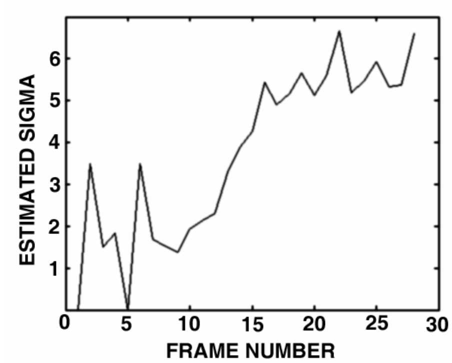
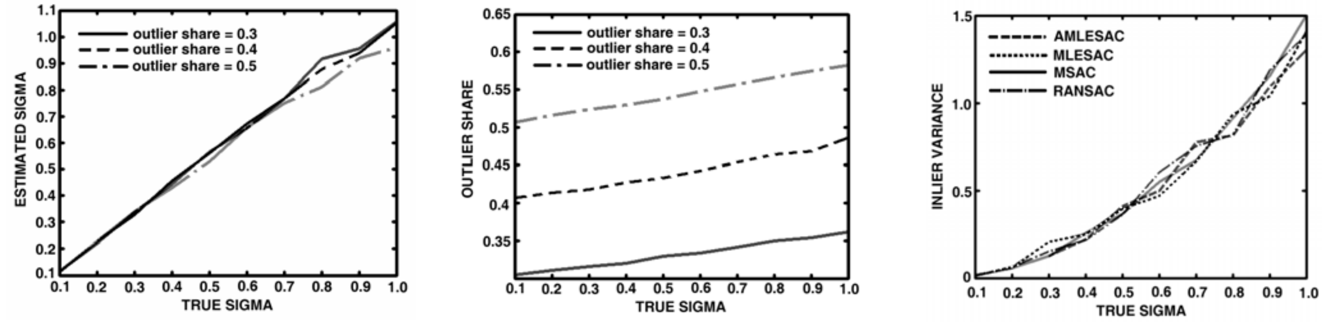


Figure 3 Estimated inlier noise σ deviation during camera calibration for “cup” image sequence

On line fitting problem we have also compared our proposed method for noise deviation σ and inlier share γ estimation



(a) (b) (c)

Figure 4 AMLESAC evaluation on line fitting problem (a) Estimated noise deviation σ of inliers plotted against true value. (b) Estimated outlier share $1-\gamma$ plotted against noise deviation σ (c) Inlier variance σ^2 of 4 robust estimators for line fitting problem: RANSAC, MLESAC, MSAC, AMLESAC plotted against true deviation of inlier noise σ. Each value was calculated by averaging the results of 100 tests. (a) and (b) were computed for outlier shares 0.3, 0.4, 0.5. Comparison (c) is demonstrated for outlier share 0.5

with one of Feng and Hung. Both methods have been demonstrated similar precision of γ and σ estimation, however the latter has lower computational complexity.

A popular measure for robust estimator accuracy comparison is inlier variance σ^2 before non-linear minimization step [4],[5]. It demonstrates the discrepancy between best hypothesis θ and points of x that has been marked as inliers. Using this measure the comparison was made between AMLESAC and other robust estimators RANSAC, MSAC and MLESAC. Figure 4 (c) shows that for line fitting all robust estimators demonstrate roughly the same estimation precision. It must be emphasized that true values for γ and σ were used for RANSAC, MSAC and MLESAC. This demonstrates that precision of γ and σ estimation in AMLESAC is sufficient to achieve the same accuracy as other robust estimator.

7.2 Synthetic data tests – pose estimation

The pose estimation task is the one of robust estimation problems, which clearly reveals the changes of noise level during camera motion estimation in one image sequence. The synthetic tests were supposed to demonstrate the ability of AMLESAC to correctly determine both the parameters of noise and outliers share while maintaining the accuracy of other robust estimator.

Each test data set consists of 200 randomly generated 3d points in 10 focal lengths away from camera center and their corresponding reprojections onto camera plane. The projection points are perturbed by Gaussian noise with zero mean and different deviation. A set of outliers is added to lower inlier share γ to 0.5. Figure 5 (a) and (b) show the estimated inlier noise variation σ^2 and outlier share $1-\gamma$ plotted against true noise variation respectively.

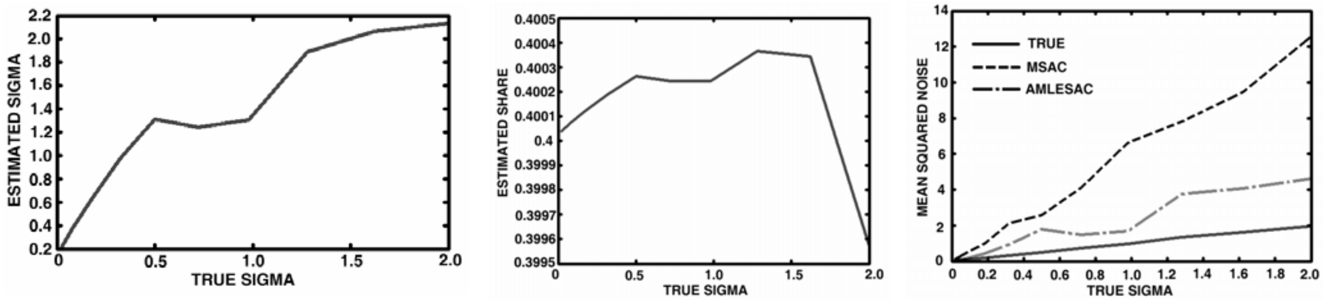
In Figure 5 (c) the comparison of pose estimation with MSAC and AMLESAC estimators is demonstrated. It can be seen that AMLESAC has generally lower mean inlier error then that of MSAC.

The AMLESAC exploits both accurate scoring function based on hypothesis likelihood and local optimization step, which combined give it superior performance compared with other robust estimators, especially in real tests, however in expense of higher computational complexity.

8. CONCLUSION

In this paper a new general robust estimation method AMLESAC has been developed. It has been demonstrated that AMLESAC can be efficiently used in such task as camera pose estimation when inlier error deviation varies from frame to frame and prohibits the application of other robust estimators. It has been shown to provide equal or superior precision compared to existing robust estimators without relying on predefined noise parameters. This show that likelihood maximization with simultaneous inlier noise σ and inlier share λ estimation, boosted by local optimization lead to significantly more accurate and robust hypotheses scoring. When the computational complexity is not a first priority the AMLESAC is preferable to all other currently existing robust estimators.

Our method differs from existing methods in several ways. First, during the robust initialization step we search for a model with maximum likelihood with simultaneous estimation of the unknown parameters σ and λ in a mixture model of errors for inliers and outliers. Second, only the subset of points is used for mixture parameters estimation and hypothesizes are locally optimized to increase precision of inlier noise deviation σ and inlier share λ estimation. Without local optimization estimated values of mixture parameters can deviate very far from true ones that leads to constant estimation failures. Third, during the non-linear refinement step we evaluate the likelihood of the model more precise then other methods by simultaneous estimation of inlier portion γ and deviation σ or by optimization in space of all model parameters including γ and σ.



(a) (b) (c)

Figure 5 AMLESAC evaluation on pose estimation problem. (a) Inlier error by AMLESAC plotted against true value. (b) Outlier share estimated by AMLESAC plotted against true inlier error variation. (c) Mean inlier variance σ^2 of hypothesis, estimated by MSAC and AMLESAC estimators plotted against true inlier error deviation. Plots and comparison are demonstrated for inlire share $\gamma = 0.5$

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